

Mathematica 11.3 Integration Test Results

Test results for the 371 problems in "7.1.5 Inverse hyperbolic sine functions.m"

Problem 1: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSinh}[c x]}{d + e x} dx$$

Optimal (type 4, 170 leaves, 8 steps) :

$$\begin{aligned} & -\frac{\text{ArcSinh}[c x]^2}{2 e} + \frac{\frac{\text{ArcSinh}[c x] \log \left[1 + \frac{e e^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}\right]}{e}}{e} + \\ & \frac{\frac{\text{ArcSinh}[c x] \log \left[1 + \frac{e e^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}\right]}{e}}{e} + \frac{\text{PolyLog}\left[2, -\frac{e e^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}\right]}{e} + \frac{\text{PolyLog}\left[2, -\frac{e e^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}\right]}{e} \end{aligned}$$

Result (type 4, 447 leaves) :

$$\begin{aligned}
& \frac{1}{8 e} \left(\pi^2 - 4 i \pi \operatorname{ArcSinh}[c x] - 4 \operatorname{ArcSinh}[c x]^2 - \right. \\
& 32 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(c d + i e) \operatorname{Cot}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{c^2 d^2 + e^2}}\right] + \\
& 4 i \pi \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] + \\
& 16 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] + \\
& 8 \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] + \\
& 4 i \pi \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] - \\
& 16 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] + \\
& 8 \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] - 4 i \pi \operatorname{Log}[c d + c e x] + \\
& \left. 8 \operatorname{PolyLog}\left[2, \frac{(c d - \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] + 8 \operatorname{PolyLog}\left[2, \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e}\right] \right)
\end{aligned}$$

Problem 2: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSinh}[c x]^2}{d + e x} dx$$

Optimal (type 4, 260 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\text{ArcSinh}[c x]^3}{3 e} + \frac{\text{ArcSinh}[c x]^2 \log \left[1 + \frac{e e^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}} \right]}{e} + \frac{\text{ArcSinh}[c x]^2 \log \left[1 + \frac{e e^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}} \right]}{e} + \\
& \frac{2 \text{ArcSinh}[c x] \text{PolyLog}[2, - \frac{e e^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}]}{e} + \frac{2 \text{ArcSinh}[c x] \text{PolyLog}[2, - \frac{e e^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}]}{e} - \\
& \frac{2 \text{PolyLog}[3, - \frac{e e^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}]}{e} - \frac{2 \text{PolyLog}[3, - \frac{e e^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}]}{e}
\end{aligned}$$

Result (type 4, 1061 leaves) :

$$\begin{aligned}
& - \frac{1}{3 e} \left(\text{ArcSinh}[c x]^3 + 24 \text{ArcSin} \left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \text{ArcSinh}[c x] \right. \\
& \left. \text{ArcTan} \left[\frac{(c d + i e) \cot \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x]) \right]}{\sqrt{c^2 d^2 + e^2}} \right] - 24 \text{ArcSin} \left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \right. \\
& \left. \text{ArcSinh}[c x] \text{ArcTan} \left[\left((c d + i e) \left(\cosh \left[\frac{1}{2} \text{ArcSinh}[c x] \right] - i \sinh \left[\frac{1}{2} \text{ArcSinh}[c x] \right] \right) \right) / \right. \right. \\
& \left. \left. \left(\sqrt{c^2 d^2 + e^2} \left(\cosh \left[\frac{1}{2} \text{ArcSinh}[c x] \right] + i \sinh \left[\frac{1}{2} \text{ArcSinh}[c x] \right] \right) \right) \right] - 3 \text{ArcSinh}[c x]^2 \right. \\
& \left. \log \left[1 + \frac{e e^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}} \right] - 3 i \pi \text{ArcSinh}[c x] \log \left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\text{ArcSinh}[c x]}}{e} \right] - \right. \\
& \left. 12 i \text{ArcSin} \left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \text{ArcSinh}[c x] \log \left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\text{ArcSinh}[c x]}}{e} \right] - \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \log \left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\text{ArcSinh}[c x]}}{e} \right] - \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \log \left[1 + \frac{e e^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}} \right] - \right. \\
& \left. 3 i \pi \text{ArcSinh}[c x] \log \left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\text{ArcSinh}[c x]}}{e} \right] + \right. \\
& \left. 12 i \text{ArcSin} \left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \text{ArcSinh}[c x] \log \left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\text{ArcSinh}[c x]}}{e} \right] - \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \log \left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\text{ArcSinh}[c x]}}{e} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \\
& 12 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \\
& 3 \operatorname{ArcSinh}[c x]^2 \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \\
& 3 \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] - \\
& 12 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c d}{e}}}{\sqrt{2}}\right] \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \\
& 3 \operatorname{ArcSinh}[c x]^2 \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] - \\
& 6 \operatorname{ArcSinh}[c x] \operatorname{PolyLog}\left[2, \frac{e e^{\operatorname{ArcSinh}[c x]}}{-c d + \sqrt{c^2 d^2 + e^2}}\right] - \\
& 6 \operatorname{ArcSinh}[c x] \operatorname{PolyLog}\left[2, -\frac{e e^{\operatorname{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}\right] + \\
& 6 \operatorname{PolyLog}\left[3, \frac{e e^{\operatorname{ArcSinh}[c x]}}{-c d + \sqrt{c^2 d^2 + e^2}}\right] + 6 \operatorname{PolyLog}\left[3, -\frac{e e^{\operatorname{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}\right]
\end{aligned}$$

Problem 3: Unable to integrate problem.

$$\int \frac{\operatorname{ArcSinh}[c x]^3}{d + e x} dx$$

Optimal (type 4, 348 leaves, 12 steps):

$$\begin{aligned}
& - \frac{\text{ArcSinh}[c x]^4}{4 e} + \frac{\text{ArcSinh}[c x]^3 \log \left[1 + \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}} \right]}{e} + \frac{\text{ArcSinh}[c x]^3 \log \left[1 + \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}} \right]}{e} + \\
& \frac{3 \text{ArcSinh}[c x]^2 \text{PolyLog}[2, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}]}{e} + \frac{3 \text{ArcSinh}[c x]^2 \text{PolyLog}[2, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}]}{e} - \\
& \frac{6 \text{ArcSinh}[c x] \text{PolyLog}[3, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}]}{e} - \frac{6 \text{ArcSinh}[c x] \text{PolyLog}[3, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}]}{e} + \\
& \frac{6 \text{PolyLog}[4, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}]}{e} + \frac{6 \text{PolyLog}[4, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}]}{e}
\end{aligned}$$

Result (type 8, 16 leaves) :

$$\int \frac{\text{ArcSinh}[c x]^3}{d + e x} dx$$

Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \text{ArcSinh}[c x]}{d + e x} dx$$

Optimal (type 4, 187 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{(a + b \text{ArcSinh}[c x])^2}{2 b e} + \frac{(a + b \text{ArcSinh}[c x]) \log \left[1 + \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}} \right]}{e} + \\
& \frac{(a + b \text{ArcSinh}[c x]) \log \left[1 + \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}} \right]}{e} + \\
& \frac{b \text{PolyLog}[2, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}]}{e} + \frac{b \text{PolyLog}[2, - \frac{e \text{e}^{\text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}]}{e}
\end{aligned}$$

Result (type 4, 460 leaves) :

$$\begin{aligned}
& \frac{a \operatorname{Log}[d + e x]}{e} + \frac{1}{8 e} b \left(\pi^2 - 4 i \pi \operatorname{ArcSinh}[c x] - 4 \operatorname{ArcSinh}[c x]^2 - \right. \\
& 32 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \operatorname{ArcTan}\left[\frac{(c d + i e) \operatorname{Cot}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{c^2 d^2 + e^2}} \right] + \\
& 4 i \pi \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] + \\
& 16 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] + 8 \operatorname{ArcSinh}[c x] \\
& \operatorname{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] + 4 i \pi \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] - \\
& 16 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}} \right] \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] + \\
& 8 \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] - 4 i \pi \operatorname{Log}[c d + c e x] + \\
& \left. 8 \operatorname{PolyLog}\left[2, \frac{(c d - \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] + 8 \operatorname{PolyLog}\left[2, \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{ArcSinh}[c x]}}{e} \right] \right)
\end{aligned}$$

Problem 16: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{d + e x} dx$$

Optimal (type 4, 291 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(a+b \operatorname{ArcSinh}[cx])^3}{3 b e} + \frac{(a+b \operatorname{ArcSinh}[cx])^2 \operatorname{Log}\left[1+\frac{e e^{\operatorname{ArcSinh}[cx]}}{c d-\sqrt{c^2 d^2+e^2}}\right]}{e} + \\
& \frac{(a+b \operatorname{ArcSinh}[cx])^2 \operatorname{Log}\left[1+\frac{e e^{\operatorname{ArcSinh}[cx]}}{c d+\sqrt{c^2 d^2+e^2}}\right]}{e} + \frac{2 b (a+b \operatorname{ArcSinh}[cx]) \operatorname{PolyLog}\left[2,-\frac{e e^{\operatorname{ArcSinh}[cx]}}{c d-\sqrt{c^2 d^2+e^2}}\right]}{e} + \\
& \frac{2 b (a+b \operatorname{ArcSinh}[cx]) \operatorname{PolyLog}\left[2,-\frac{e e^{\operatorname{ArcSinh}[cx]}}{c d+\sqrt{c^2 d^2+e^2}}\right]}{e} - \\
& \frac{2 b^2 \operatorname{PolyLog}\left[3,-\frac{e e^{\operatorname{ArcSinh}[cx]}}{c d-\sqrt{c^2 d^2+e^2}}\right]}{e} - \frac{2 b^2 \operatorname{PolyLog}\left[3,-\frac{e e^{\operatorname{ArcSinh}[cx]}}{c d+\sqrt{c^2 d^2+e^2}}\right]}{e}
\end{aligned}$$

Result (type 4, 1521 leaves):

$$\begin{aligned}
& \frac{1}{12 e} \left(12 a^2 \operatorname{Log}[d+e x] + 3 a b \left(\pi^2 - 4 \pm \pi \operatorname{ArcSinh}[cx] - 4 \operatorname{ArcSinh}[cx]^2 - \right. \right. \\
& \left. \left. 32 \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c d}{e}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(c d+i e) \operatorname{Cot}\left[\frac{1}{4}(\pi+2 \pm \operatorname{ArcSinh}[cx])\right]}{\sqrt{c^2 d^2+e^2}}\right] + \right. \\
& \left. 4 \pm \pi \operatorname{Log}\left[1+\frac{(-c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] + \right. \\
& \left. 16 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c d}{e}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{(-c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] + 8 \operatorname{ArcSinh}[cx] \right. \\
& \left. \operatorname{Log}\left[1+\frac{(-c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] + 4 \pm \pi \operatorname{Log}\left[1-\frac{(c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] - \right. \\
& \left. 16 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i c d}{e}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{(c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] + \right. \\
& \left. 8 \operatorname{ArcSinh}[cx] \operatorname{Log}\left[1-\frac{(c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] - \right. \\
& \left. 4 \pm \pi \operatorname{Log}\left[c (d+e x)\right] + 8 \operatorname{PolyLog}\left[2,\frac{(c d-\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] + \right. \\
& \left. 8 \operatorname{PolyLog}\left[2,\frac{(c d+\sqrt{c^2 d^2+e^2}) e^{\operatorname{ArcSinh}[cx]}}{e}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& 4 b^2 \left(\text{ArcSinh}[c x]^3 + 24 \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \text{ArcSinh}[c x] \right. \\
& \left. \text{ArcTan}\left[\frac{(c d + i e) \text{Cot}\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]}{\sqrt{c^2 d^2 + e^2}}\right] - 24 \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right]\right. \\
& \left. \text{ArcSinh}[c x] \text{ArcTan}\left[\left((c d + i e) \left(\text{Cosh}\left[\frac{1}{2} \text{ArcSinh}[c x]\right] - i \text{Sinh}\left[\frac{1}{2} \text{ArcSinh}[c x]\right]\right)\right)\right.\right. \\
& \left. \left. \left(\sqrt{c^2 d^2 + e^2} \left(\text{Cosh}\left[\frac{1}{2} \text{ArcSinh}[c x]\right] + i \text{Sinh}\left[\frac{1}{2} \text{ArcSinh}[c x]\right]\right)\right)\right] - 3 \text{ArcSinh}[c x]^2\right. \\
& \left. \text{Log}\left[1 + \frac{e^{i \text{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2 + e^2}}\right] - 3 i \pi \text{ArcSinh}[c x] \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{i \text{ArcSinh}[c x]}}{e}\right] - \right. \\
& \left. 12 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \text{ArcSinh}[c x] \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{i \text{ArcSinh}[c x]}}{e}\right] - \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) e^{i \text{ArcSinh}[c x]}}{e}\right] - \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \text{Log}\left[1 + \frac{e^{i \text{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2 + e^2}}\right] - \right. \\
& \left. 3 i \pi \text{ArcSinh}[c x] \text{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{i \text{ArcSinh}[c x]}}{e}\right] + \right. \\
& \left. 12 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \text{ArcSinh}[c x] \text{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{i \text{ArcSinh}[c x]}}{e}\right] - \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \text{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) e^{i \text{ArcSinh}[c x]}}{e}\right] + \right. \\
& \left. 3 i \pi \text{ArcSinh}[c x] \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \right. \\
& \left. 12 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i c d}{e}}}{\sqrt{2}}\right] \text{ArcSinh}[c x] \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \right. \\
& \left. 3 \text{ArcSinh}[c x]^2 \text{Log}\left[1 + \frac{(-c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] + \right. \\
& \left. 3 i \pi \text{ArcSinh}[c x] \text{Log}\left[1 - \frac{(c d + \sqrt{c^2 d^2 + e^2}) (c x + \sqrt{1 + c^2 x^2})}{e}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 12 \ln \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{icd}{e}}}{\sqrt{2}} \right] \operatorname{ArcSinh}[cx] \operatorname{Log} \left[1 - \frac{(cd + \sqrt{c^2 d^2 + e^2}) (cx + \sqrt{1 + c^2 x^2})}{e} \right] + \\
& 3 \operatorname{ArcSinh}[cx]^2 \operatorname{Log} \left[1 - \frac{(cd + \sqrt{c^2 d^2 + e^2}) (cx + \sqrt{1 + c^2 x^2})}{e} \right] - 6 \operatorname{ArcSinh}[cx] \\
& \operatorname{PolyLog} \left[2, \frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{-cd + \sqrt{c^2 d^2 + e^2}} \right] - 6 \operatorname{ArcSinh}[cx] \operatorname{PolyLog} \left[2, -\frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{cd + \sqrt{c^2 d^2 + e^2}} \right] + \\
& \left. \left. \left. 6 \operatorname{PolyLog} \left[3, \frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{-cd + \sqrt{c^2 d^2 + e^2}} \right] + 6 \operatorname{PolyLog} \left[3, -\frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{cd + \sqrt{c^2 d^2 + e^2}} \right] \right) \right\}
\end{aligned}$$

Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{(d + e x)^2} dx$$

Optimal (type 4, 263 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{\left(a + b \operatorname{ArcSinh}[cx]\right)^2}{e(d + ex)} + \frac{2bc \left(a + b \operatorname{ArcSinh}[cx]\right) \operatorname{Log}\left[1 + \frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{c d - \sqrt{c^2 d^2 + e^2}}\right]}{e \sqrt{c^2 d^2 + e^2}} \\
 & + \frac{2bc \left(a + b \operatorname{ArcSinh}[cx]\right) \operatorname{Log}\left[1 + \frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{c d + \sqrt{c^2 d^2 + e^2}}\right]}{e \sqrt{c^2 d^2 + e^2}} + \\
 & \frac{2b^2 c \operatorname{PolyLog}\left[2, -\frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{c d - \sqrt{c^2 d^2 + e^2}}\right]}{e \sqrt{c^2 d^2 + e^2}} - \frac{2b^2 c \operatorname{PolyLog}\left[2, -\frac{e^{e^{\operatorname{ArcSinh}[cx]}}}{c d + \sqrt{c^2 d^2 + e^2}}\right]}{e \sqrt{c^2 d^2 + e^2}}
 \end{aligned}$$

Result (type 4, 1381 leaves):

$$-\frac{a^2}{e(d+ex)} + 2abc \left(-\frac{\text{ArcSinh}[cx]}{e(c d + c e x)} + \frac{\text{Log}[c d + c e x] - \text{Log}[e - c^2 d x + \sqrt{c^2 d^2 + e^2} \sqrt{1 + c^2 x^2}]}{e \sqrt{c^2 d^2 + e^2}} \right) + b^2 c \left(-\frac{\text{ArcSinh}[cx]^2}{e(c d + c e x)} + \frac{1}{e} 2 \left(-\frac{i \pi \text{ArcTanh}\left[\frac{-e+c d \tanh\left[\frac{1}{2} \text{ArcSinh}[cx]\right]}{\sqrt{c^2 d^2+e^2}}\right]}{\sqrt{c^2 d^2+e^2}} - \frac{1}{\sqrt{-c^2 d^2-e^2}} \left(2 \left(\frac{\pi}{2} - i \text{ArcSinh}[cx]\right) \text{ArcTanh}\left[\frac{(c d - i e) \cot\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[cx]\right)\right]}{\sqrt{-c^2 d^2-e^2}}\right] - \right. \right. \right)$$

$$\begin{aligned}
& 2 \operatorname{ArcCos}\left[-\frac{\frac{i}{2} c d}{e}\right] \operatorname{ArcTanh}\left[\frac{(-c d - \frac{i}{2} e) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{\frac{i}{2} c d}{e}\right] - 2 \frac{i}{2} \left(\operatorname{ArcTanh}\left[\frac{(c d - \frac{i}{2} e) \operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right] - \right. \right. \\
& \left. \left. \operatorname{ArcTanh}\left[\frac{(-c d - \frac{i}{2} e) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right]\right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-c^2 d^2 - e^2} e^{-\frac{1}{2} \frac{i}{2} \left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)}}{\sqrt{2} \sqrt{-\frac{i}{2} e} \sqrt{c d + c e x}}\right] + \left(\operatorname{ArcCos}\left[-\frac{\frac{i}{2} c d}{e}\right] + \right. \\
& 2 \frac{i}{2} \left(\operatorname{ArcTanh}\left[\frac{(c d - \frac{i}{2} e) \operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right] - \right. \\
& \left. \left. \operatorname{ArcTanh}\left[\frac{(-c d - \frac{i}{2} e) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right]\right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-c^2 d^2 - e^2} e^{\frac{1}{2} \frac{i}{2} \left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)}}{\sqrt{2} \sqrt{-\frac{i}{2} e} \sqrt{c d + c e x}}\right] - \left(\operatorname{ArcCos}\left[-\frac{\frac{i}{2} c d}{e}\right] + \right. \\
& 2 \frac{i}{2} \operatorname{ArcTanh}\left[\frac{(-c d - \frac{i}{2} e) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right] \operatorname{Log}[1 - \\
& \left(\frac{i}{2} \left(c d - \frac{i}{2} \sqrt{-c^2 d^2 - e^2} \right) \left(c d - \frac{i}{2} e - \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) / \\
& \left(e \left(c d - \frac{i}{2} e + \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) + \\
& \left(-\operatorname{ArcCos}\left[-\frac{\frac{i}{2} c d}{e}\right] + 2 \frac{i}{2} \operatorname{ArcTanh}\left[\frac{(-c d - \frac{i}{2} e) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 d^2 - e^2}}\right] \right) \operatorname{Log}[1 - \\
& \left(\frac{i}{2} \left(c d + \frac{i}{2} \sqrt{-c^2 d^2 - e^2} \right) \left(c d - \frac{i}{2} e - \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) / \\
& \left(e \left(c d - \frac{i}{2} e + \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) + \\
& \frac{i}{2} \left(\operatorname{PolyLog}[2, \left(\frac{i}{2} \left(c d - \frac{i}{2} \sqrt{-c^2 d^2 - e^2} \right) \left(c d - \frac{i}{2} e - \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) / \left(e \left(c d - \frac{i}{2} e + \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) - \operatorname{PolyLog}[2, \left(\frac{i}{2} \left(c d + \frac{i}{2} \sqrt{-c^2 d^2 - e^2} \right) \left(c d - \frac{i}{2} e - \sqrt{-c^2 d^2 - e^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - \frac{i}{2} \operatorname{ArcSinh}[c x]\right)\right] \right) \right) /
\end{aligned}$$

$$\left. \left(e \left(c d - i e + \sqrt{-c^2 d^2 - e^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right)$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{(d + e x)^3} dx$$

Optimal (type 4, 349 leaves, 13 steps):

$$\begin{aligned} & -\frac{b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])}{(c^2 d^2+e^2) (d+e x)} - \frac{(a+b \operatorname{ArcSinh}[c x])^2}{2 e (d+e x)^2} + \\ & \frac{b c^3 d (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log} \left[1 + \frac{e e^{\operatorname{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2+e^2}} \right]}{e (c^2 d^2+e^2)^{3/2}} - \frac{b c^3 d (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log} \left[1 + \frac{e e^{\operatorname{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2+e^2}} \right]}{e (c^2 d^2+e^2)^{3/2}} + \\ & \frac{b^2 c^2 \operatorname{Log}[d+e x]}{e (c^2 d^2+e^2)} + \frac{b^2 c^3 d \operatorname{PolyLog}[2, -\frac{e e^{\operatorname{ArcSinh}[c x]}}{c d - \sqrt{c^2 d^2+e^2}}]}{e (c^2 d^2+e^2)^{3/2}} - \frac{b^2 c^3 d \operatorname{PolyLog}[2, -\frac{e e^{\operatorname{ArcSinh}[c x]}}{c d + \sqrt{c^2 d^2+e^2}}]}{e (c^2 d^2+e^2)^{3/2}} \end{aligned}$$

Result (type 4, 1558 leaves):

$$\begin{aligned} & -\frac{a^2}{2 e (d+e x)^2} + \\ & 2 a b c^2 \left(-\frac{\operatorname{ArcSinh}[c x]}{2 e (c d + c e x)^2} + \left(-e \sqrt{c^2 d^2 + e^2} \sqrt{1+c^2 x^2} + c d (c d + c e x) \operatorname{Log}[c d + c e x] - \right. \right. \\ & \left. \left. c d (c d + c e x) \operatorname{Log}[e - c^2 d x + \sqrt{c^2 d^2 + e^2} \sqrt{1+c^2 x^2}] \right) \right) / \\ & \left(2 e (-i c d + e) (i c d + e) \sqrt{c^2 d^2 + e^2} (c d + c e x) \right) + \\ & b^2 c^2 \left(-\frac{\sqrt{1+c^2 x^2} \operatorname{ArcSinh}[c x]}{(c^2 d^2+e^2) (c d + c e x)} - \frac{\operatorname{ArcSinh}[c x]^2}{2 e (c d + c e x)^2} + \frac{\operatorname{Log}[1 + \frac{e x}{d}]}{e (c^2 d^2+e^2)} + \right. \\ & \left. \frac{1}{e (c^2 d^2+e^2)} c d \left(-\frac{i \pi \operatorname{ArcTanh} \left[\frac{-e+c d \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcSinh}[c x] \right]}{\sqrt{c^2 d^2+e^2}} \right]}{\sqrt{c^2 d^2+e^2}} - \right. \right. \end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{-c^2 d^2 - e^2}} \left(2 \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \operatorname{ArcTanh} \left[\frac{(c d - i e) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] - \right. \\
& \quad 2 \operatorname{ArcCos} \left[-\frac{i c d}{e} \right] \operatorname{ArcTanh} \left[\frac{(-c d - i e) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] + \\
& \quad \left. \left(\operatorname{ArcCos} \left[-\frac{i c d}{e} \right] - 2 i \left(\operatorname{ArcTanh} \left[\frac{(c d - i e) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(-c d - i e) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] \right) \right) \right) \\
& \quad \operatorname{Log} \left[\frac{\sqrt{-c^2 d^2 - e^2} e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right)}}{\sqrt{2} \sqrt{-i e} \sqrt{c d + c e x}} \right] + \left(\operatorname{ArcCos} \left[-\frac{i c d}{e} \right] + \right. \\
& \quad 2 i \left(\operatorname{ArcTanh} \left[\frac{(c d - i e) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] - \right. \\
& \quad \left. \left. \operatorname{ArcTanh} \left[\frac{(-c d - i e) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] \right) \right) \\
& \quad \operatorname{Log} \left[\frac{\sqrt{-c^2 d^2 - e^2} e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right)}}{\sqrt{2} \sqrt{-i e} \sqrt{c d + c e x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{i c d}{e} \right] + \right. \\
& \quad \left. \left. 2 i \operatorname{ArcTanh} \left[\frac{(-c d - i e) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] \right) \operatorname{Log} [1 - \right. \\
& \quad \left. \left(i \left(c d - i \sqrt{-c^2 d^2 - e^2} \right) \left(c d - i e - \sqrt{-c^2 d^2 - e^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \right. \\
& \quad \left. \left(e \left(c d - i e + \sqrt{-c^2 d^2 - e^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) + \right. \\
& \quad \left. \left(-\operatorname{ArcCos} \left[-\frac{i c d}{e} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-c d - i e) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 d^2 - e^2}} \right] \right) \operatorname{Log} [1 - \right. \\
& \quad \left. \left(i \left(c d + i \sqrt{-c^2 d^2 - e^2} \right) \left(c d - i e - \sqrt{-c^2 d^2 - e^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \right. \\
& \quad \left. \left(e \left(c d - i e + \sqrt{-c^2 d^2 - e^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) + \right. \\
& \quad \left. \left(i \left(\operatorname{PolyLog} [2, \left(i \left(c d - i \sqrt{-c^2 d^2 - e^2} \right) \left(c d - i e - \sqrt{-c^2 d^2 - e^2} \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \left(e \left(c d - i e + \sqrt{-c^2 d^2 - e^2} \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right) - \operatorname{PolyLog} [2, \left(i \left(c d + i \sqrt{-c^2 d^2 - e^2} \right) \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right]
\end{aligned}$$

$$\left(\frac{\left(c d - i e - \sqrt{-c^2 d^2 - e^2} \right) \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]}{e \left(c d - i e + \sqrt{-c^2 d^2 - e^2} \right) \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right]} \right) \right)$$

Problem 31: Unable to integrate problem.

$$\int (d + e x)^m (a + b \operatorname{ArcSinh}[c x]) dx$$

Optimal (type 6, 179 leaves, 3 steps):

$$-\left(\left(b c (d + e x)^{2+m} \sqrt{1 - \frac{d + e x}{d - \frac{e}{\sqrt{-c^2}}}} \right. \right. \\ \left. \left. - \sqrt{1 - \frac{d + e x}{d + \frac{e}{\sqrt{-c^2}}}} \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{1}{2}, 3+m, \frac{d + e x}{d - \frac{e}{\sqrt{-c^2}}}, \frac{d + e x}{d + \frac{e}{\sqrt{-c^2}}} \right] \right) \right) \\ \left. \left(e^2 (1+m) (2+m) \sqrt{1 + c^2 x^2} \right) + \frac{(d + e x)^{1+m} (a + b \operatorname{ArcSinh}[c x])}{e (1+m)} \right)$$

Result (type 8, 18 leaves):

$$\int (d + e x)^m (a + b \operatorname{ArcSinh}[c x]) dx$$

Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{f + g x} dx$$

Optimal (type 4, 664 leaves, 22 steps):

$$\begin{aligned}
& \frac{a \sqrt{d + c^2 d x^2}}{g} - \frac{b c x \sqrt{d + c^2 d x^2}}{g \sqrt{1 + c^2 x^2}} + \frac{b \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{g} - \\
& \frac{c x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b g \sqrt{1 + c^2 x^2}} - \frac{\left(1 + \frac{c^2 f^2}{g^2}\right) \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c (f + g x) \sqrt{1 + c^2 x^2}} + \\
& \frac{\sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c (f + g x)} - \\
& \frac{a \sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2} \operatorname{ArcTanh}\left[\frac{g - c^2 f x}{\sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}}\right]}{g^2 \sqrt{1 + c^2 x^2}} + \\
& \frac{b \sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{g^2 \sqrt{1 + c^2 x^2}} - \\
& \frac{b \sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{g^2 \sqrt{1 + c^2 x^2}} + \\
& \frac{b \sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2} \operatorname{PolyLog}\left[2, - \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{g^2 \sqrt{1 + c^2 x^2}} - \\
& \frac{b \sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2} \operatorname{PolyLog}\left[2, - \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{g^2 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 1552 leaves) :

$$\begin{aligned}
& \frac{a \sqrt{d (1 + c^2 x^2)}}{g} + \frac{a \sqrt{d} \sqrt{c^2 f^2 + g^2} \operatorname{Log}[f + g x]}{g^2} - \frac{a c \sqrt{d} f \operatorname{Log}[c d x + \sqrt{d} \sqrt{d (1 + c^2 x^2)}]}{g^2} - \\
& \frac{a \sqrt{d} \sqrt{c^2 f^2 + g^2} \operatorname{Log}[d g - c^2 d f x + \sqrt{d} \sqrt{c^2 f^2 + g^2} \sqrt{d (1 + c^2 x^2)}]}{g^2} + \\
& b \left(- \frac{c x \sqrt{d (1 + c^2 x^2)}}{g \sqrt{1 + c^2 x^2}} + \frac{\sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]}{g} - \frac{c f \sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]^2}{2 g^2 \sqrt{1 + c^2 x^2}} + \right. \\
& \left. \frac{1}{g^2 \sqrt{1 + c^2 x^2}} (c^2 f^2 + g^2) \sqrt{d (1 + c^2 x^2)} \left(- \frac{\frac{1}{2} \pi \operatorname{ArcTanh}\left[\frac{-g + c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2}} - \right. \right. \\
& \left. \left. \frac{1}{\sqrt{c^2 f^2 + g^2}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \quad 2 \operatorname{ArcCos} \left[-\frac{i c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(-c f - i g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \\
& \quad \left. \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] - 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(-c f - i g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \right) \\
& \operatorname{Log} \left[\frac{e^{-\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right)} \sqrt{-c^2 f^2 - g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f + c g x}} \right] + \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& \quad 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \quad \left. \left. \operatorname{ArcTanh} \left[\frac{(-c f - i g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right)} \sqrt{-c^2 f^2 - g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& \quad \left. 2 i \operatorname{ArcTanh} \left[\frac{(-c f - i g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} [1 - \\
& \quad \left(i \left(c f - i \sqrt{-c^2 f^2 - g^2} \right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right) / \\
& \quad \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right) + \\
& \quad \left(-\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-c f - i g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} [1 - \\
& \quad \left(i \left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right) / \\
& \quad \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right) + \\
& \quad i \left(\operatorname{PolyLog} [2, \left(i \left(c f - i \sqrt{-c^2 f^2 - g^2} \right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right) / \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right) - \operatorname{PolyLog} [2, \left(i \left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[cx] \right) \right] \right) \right)
\end{aligned}$$

$$\left(\left(c f - i g - \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \\ \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right)$$

Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(f + g x)^2} dx$$

Optimal (type 4, 781 leaves, 35 steps):

$$\begin{aligned} & -\frac{a \sqrt{d + c^2 d x^2}}{g (f + g x)} - \frac{b \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{g (f + g x)} + \frac{a c^3 f^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{g^2 (c^2 f^2 + g^2) \sqrt{1 + c^2 x^2}} + \\ & \frac{b c^3 f^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]^2}{2 g^2 (c^2 f^2 + g^2) \sqrt{1 + c^2 x^2}} - \frac{(g - c^2 f x)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c (c^2 f^2 + g^2) (f + g x)^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{\sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c (f + g x)^2} + \frac{a c^2 f \sqrt{d + c^2 d x^2} \operatorname{ArcTanh} \left[\frac{g - c^2 f x}{\sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}} \right]}{g^2 \sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}} - \\ & \frac{b c^2 f \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log} \left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}} \right]}{g^2 \sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}} + \\ & \frac{b c^2 f \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log} \left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}} \right]}{g^2 \sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}} + \frac{b c \sqrt{d + c^2 d x^2} \operatorname{Log}[f + g x]}{g^2 \sqrt{1 + c^2 x^2}} - \\ & \frac{b c^2 f \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}]}{g^2 \sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}} + \frac{b c^2 f \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}]}{g^2 \sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}} \end{aligned}$$

Result (type 4, 1574 leaves):

$$\begin{aligned} & -\frac{a \sqrt{d (1 + c^2 x^2)}}{g (f + g x)} - \frac{a c^2 \sqrt{d} f \operatorname{Log}[f + g x]}{g^2 \sqrt{c^2 f^2 + g^2}} + \frac{a c \sqrt{d} \operatorname{Log}[c d x + \sqrt{d} \sqrt{d (1 + c^2 x^2)}]}{g^2} + \\ & \frac{a c^2 \sqrt{d} f \operatorname{Log}[d g - c^2 d f x + \sqrt{d} \sqrt{c^2 f^2 + g^2} \sqrt{d (1 + c^2 x^2)}]}{g^2 \sqrt{c^2 f^2 + g^2}} + \end{aligned}$$

$$\begin{aligned}
& b c \left(-\frac{\sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]}{g (c f + c g x)} + \frac{\sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]^2}{2 g^2 \sqrt{1 + c^2 x^2}} + \frac{\sqrt{d (1 + c^2 x^2)} \operatorname{Log}\left[1 + \frac{g x}{f}\right]}{g^2 \sqrt{1 + c^2 x^2}} - \right. \\
& \left. \frac{1}{g^2 \sqrt{1 + c^2 x^2}} c f \sqrt{d (1 + c^2 x^2)} \left(-\frac{i \pi \operatorname{ArcTanh}\left[\frac{-g+c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2+g^2}}\right]}{\sqrt{c^2 f^2+g^2}} - \right. \right. \\
& \left. \left. \frac{1}{\sqrt{-c^2 f^2-g^2}} \left(2 \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right) \operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \\
& \left. \left. 2 \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f-i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right. \right. \\
& \left. \left. \operatorname{ArcCos}\left[-\frac{i c f}{g}\right]-2 i \left(\operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f-i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)} \sqrt{-c^2 f^2-g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f+c g x}}\right] + \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \\
& \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f-i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)} \sqrt{-c^2 f^2-g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f+c g x}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(-c f-i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \operatorname{Log}\left[1 - \right. \\
& \left. \left(i \left(c f-i \sqrt{-c^2 f^2-g^2} \right) \left(c f-i g-\sqrt{-c^2 f^2-g^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right] \right) \right) / \\
& \left. \left(g \left(c f-i g+\sqrt{-c^2 f^2-g^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right] \right) \right) + \\
& \left. \left(-\operatorname{ArcCos}\left[-\frac{i c f}{g}\right]+2 i \operatorname{ArcTanh}\left[\frac{(-c f-i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2}-i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \operatorname{Log}\left[1 - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{i}{2} \left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \\
& \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) + \\
& i \left(\operatorname{PolyLog}[2, \left(\frac{i}{2} \left(c f - i \sqrt{-c^2 f^2 - g^2} \right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right)] - \operatorname{PolyLog}[2, \left(\frac{i}{2} \left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \left. \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \right] \right) \right)] \right)
\end{aligned}$$

Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(d + c^2 dx^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{f + g x} dx$$

Optimal (type 4, 984 leaves, 29 steps):

$$\begin{aligned}
& \frac{a d (c^2 f^2 + g^2) \sqrt{d + c^2 d x^2}}{g^3} - \frac{b c d x \sqrt{d + c^2 d x^2}}{3 g \sqrt{1 + c^2 x^2}} - \frac{b c d (c^2 f^2 + g^2) x \sqrt{d + c^2 d x^2}}{g^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{b c^3 d f x^2 \sqrt{d + c^2 d x^2}}{4 g^2 \sqrt{1 + c^2 x^2}} - \frac{b c^3 d x^3 \sqrt{d + c^2 d x^2}}{9 g \sqrt{1 + c^2 x^2}} + \frac{b d (c^2 f^2 + g^2) \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{g^3} - \\
& \frac{c^2 d f x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{2 g^2} + \frac{d (1 + c^2 x^2) \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 g} - \\
& \frac{c d f \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{4 b g^2 \sqrt{1 + c^2 x^2}} - \frac{c d (c^2 f^2 + g^2) x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b g^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{d (c^2 f^2 + g^2)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c g^4 (f + g x) \sqrt{1 + c^2 x^2}} + \\
& \frac{d (c^2 f^2 + g^2) \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c g^2 (f + g x)} - \\
& \frac{a d (c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2} \operatorname{Arctanh}\left[\frac{g - c^2 f x}{\sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}}\right]}{g^4 \sqrt{1 + c^2 x^2}} + \\
& \frac{b d (c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{g^4 \sqrt{1 + c^2 x^2}} - \\
& \frac{b d (c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{g^4 \sqrt{1 + c^2 x^2}} + \\
& \frac{b d (c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2} \operatorname{PolyLog}\left[2, - \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{g^4 \sqrt{1 + c^2 x^2}} - \\
& \frac{b d (c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2} \operatorname{PolyLog}\left[2, - \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{g^4 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 4049 leaves):

$$\begin{aligned}
& \sqrt{d (1 + c^2 x^2)} \left(\frac{a d (3 c^2 f^2 + 4 g^2)}{3 g^3} - \frac{a c^2 d f x}{2 g^2} + \frac{a c^2 d x^2}{3 g} \right) + \\
& \frac{a d^{3/2} (c^2 f^2 + g^2)^{3/2} \operatorname{Log}[f + g x]}{g^4} - \frac{a c d^{3/2} f (2 c^2 f^2 + 3 g^2) \operatorname{Log}[c d x + \sqrt{d} \sqrt{d (1 + c^2 x^2)}]}{2 g^4} - \\
& \frac{\frac{1}{g^4} a d^{3/2} (c^2 f^2 + g^2)^{3/2} \operatorname{Log}[d g - c^2 d f x + \sqrt{d} \sqrt{c^2 f^2 + g^2} \sqrt{d (1 + c^2 x^2)}]}{+}
\end{aligned}$$

$$\begin{aligned}
& b d \left(-\frac{c x \sqrt{d (1 + c^2 x^2)}}{g \sqrt{1 + c^2 x^2}} + \frac{\sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]}{g} - \frac{c f \sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]^2}{2 g^2 \sqrt{1 + c^2 x^2}} + \right. \\
& \left. \frac{1}{g^2 \sqrt{1 + c^2 x^2}} (c^2 f^2 + g^2) \sqrt{d (1 + c^2 x^2)} \left(-\frac{\frac{i \pi \operatorname{ArcTanh}\left[\frac{-g+c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2+g^2}}\right]}{\sqrt{c^2 f^2+g^2}} - \right. \right. \\
& \left. \left. \frac{1}{\sqrt{-c^2 f^2-g^2}} \left(2 \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right) \operatorname{ArcTanh}\left[\frac{(c f - i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \\
& \left. \left. 2 \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right. \right. \\
& \left. \left. \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f - i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)} \sqrt{-c^2 f^2-g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f + c g x}}\right] + \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f - i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \\
& \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)} \sqrt{-c^2 f^2-g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f + c g x}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \operatorname{Log}\left[1 - \right. \\
& \left. \left(i \left(c f - i \sqrt{-c^2 f^2-g^2}\right) \left(c f - i g - \sqrt{-c^2 f^2-g^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]\right) \right) / \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2-g^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]\right) \right) + \\
& \left. \left(-\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \operatorname{Log}\left[1 - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\text{i}}{2} \left(c f + \frac{\text{i}}{2} \sqrt{-c^2 f^2 - g^2} \right) \left(c f - \frac{\text{i}}{2} g - \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - \frac{\text{i}}{2} \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \\
& \left(g \left(c f - \frac{\text{i}}{2} g + \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - \frac{\text{i}}{2} \operatorname{ArcSinh}[c x] \right) \right] \right) \right) + \\
& \frac{1}{2} \left(\operatorname{PolyLog}[2, \left(\frac{\text{i}}{2} \left(c f - \frac{\text{i}}{2} \sqrt{-c^2 f^2 - g^2} \right) \left(c f - \frac{\text{i}}{2} g - \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - \frac{\text{i}}{2} \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \left(g \left(c f - \frac{\text{i}}{2} g + \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - \frac{\text{i}}{2} \operatorname{ArcSinh}[c x] \right) \right] \right) \right)] - \operatorname{PolyLog}[2, \left(\frac{\text{i}}{2} \left(c f + \frac{\text{i}}{2} \sqrt{-c^2 f^2 - g^2} \right) \left(c f - \frac{\text{i}}{2} g - \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - \frac{\text{i}}{2} \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \\
& \left(g \left(c f - \frac{\text{i}}{2} g + \sqrt{-c^2 f^2 - g^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - \frac{\text{i}}{2} \operatorname{ArcSinh}[c x] \right) \right] \right) \right)] \right) + \\
& b d \left(\frac{1}{8 \sqrt{1 + c^2 x^2}} \sqrt{d (1 + c^2 x^2)} \left(\frac{\frac{\text{i}}{2} \pi \operatorname{ArcTanh} \left[\frac{-g + c f \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcSinh}[c x] \right]}{\sqrt{c^2 f^2 + g^2}} \right]}{\sqrt{c^2 f^2 + g^2}} + \right. \right. \\
& \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \operatorname{ArcCos} \left[-\frac{\frac{\text{i}}{2} c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(c f + \frac{\text{i}}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \\
& (\pi - 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh} \left[\frac{(c f - \frac{\text{i}}{2} g) \tan \left[\frac{1}{4} (\pi + 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \\
& \left. \left. \operatorname{ArcCos} \left[-\frac{\frac{\text{i}}{2} c f}{g} \right] - 2 \frac{\text{i}}{2} \operatorname{ArcTanh} \left[\frac{(c f + \frac{\text{i}}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \right. \\
& 2 \frac{\text{i}}{2} \operatorname{ArcTanh} \left[\frac{(c f - \frac{\text{i}}{2} g) \tan \left[\frac{1}{4} (\pi + 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \\
& \left. \left. \operatorname{Log} \left[\frac{\left(\frac{1}{2} - \frac{\text{i}}{2} \right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-\frac{\text{i}}{2} g} \sqrt{c f + c g x}} \right] + \right. \right. \\
& \left. \left. \operatorname{ArcCos} \left[-\frac{\frac{\text{i}}{2} c f}{g} \right] + 2 \frac{\text{i}}{2} \left(\operatorname{ArcTanh} \left[\frac{(c f + \frac{\text{i}}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - \frac{\text{i}}{2} g) \tan \left[\frac{1}{4} (\pi + 2 \frac{\text{i}}{2} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) e^{\frac{1}{2} \text{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}}\right] - \left(\text{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \quad \left. 2 i \text{ArcTanh}\left[\frac{(c f + i g) \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right]\right) \\
& \text{Log}\left[\left((i c f + g) \left(-i c f + g + \sqrt{-c^2 f^2 - g^2}\right) \left(1 + i \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right)\right] / \\
& \quad \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right] - \\
& \quad \left(\text{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \text{ArcTanh}\left[\frac{(c f + i g) \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right]\right) \\
& \text{Log}\left[\left((i c f + g) \left(i c f - g + \sqrt{-c^2 f^2 - g^2}\right) \left(i + \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right)\right] / \\
& \quad \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right] + \\
& \quad i \left(\text{PolyLog}\left[2, \left(\left(i c f + \sqrt{-c^2 f^2 - g^2}\right) \left(i c f + g - i \sqrt{-c^2 f^2 - g^2}\right.\right.\right.\right. \\
& \quad \left.\left.\left.\left.\cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right)\right] / \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2}\right.\right.\right. \\
& \quad \left.\left.\left.\cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right] - \text{PolyLog}\left[2, \left(\left(c f + i \sqrt{-c^2 f^2 - g^2}\right.\right.\right. \\
& \quad \left.\left.\left.- c f + i g + \sqrt{-c^2 f^2 - g^2} \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right)\right] / \\
& \quad \left.g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \cot\left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[c x])\right]\right)\right)\right] + \\
& \frac{1}{72 g^4 \sqrt{1 + c^2 x^2}} \sqrt{d (1 + c^2 x^2)} \left(-18 c g (4 c^2 f^2 + g^2) x + 18 g (4 c^2 f^2 + g^2) \right. \\
& \quad \left. \sqrt{1 + c^2 x^2} \text{ArcSinh}[c x] - \right. \\
& \quad \left. 18 c f (2 c^2 f^2 + g^2) \text{ArcSinh}[c x]^2 + 9 c f g^2 \cosh[2 \text{ArcSinh}[c x]] + \right. \\
& \quad \left. 6 g^3 \text{ArcSinh}[c x] \cosh[3 \text{ArcSinh}[c x]] + \right. \\
& \quad \left. 9 (8 c^4 f^4 + 8 c^2 f^2 g^2 + g^4) \left(-\frac{i \pi \text{ArcTanh}\left[\frac{-g + c f \tanh\left[\frac{1}{2} \text{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2}} - \right. \right. \\
& \quad \left. \left. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \\
& \quad \left. (\pi - 2 i \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh} \left[\frac{(c f - \frac{i}{2} g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \\
& \quad \left. \operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \quad \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f - \frac{i}{2} g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} \left[\right. \\
& \quad \left. \left(\frac{1}{2} - \frac{i}{2} \right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2} \right] + \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] + \right. \\
& \quad \left. 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \quad \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - \frac{i}{2} g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \quad \operatorname{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2} \right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-\frac{i}{2} g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] + \right. \\
& \quad \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} \left[\right. \\
& \quad \left. \left((\frac{i}{2} c f + g) \left(-\frac{i}{2} c f + g + \sqrt{-c^2 f^2 - g^2} \right) \left(1 + \frac{i}{2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] / \\
& \quad \left(g \left(\frac{i}{2} c f + g + \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) - \\
& \quad \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \\
& \quad \operatorname{Log} \left[\left((\frac{i}{2} c f + g) \left(\frac{i}{2} c f - g + \sqrt{-c^2 f^2 - g^2} \right) \left(\frac{i}{2} + \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] / \\
& \quad \left(g \left(c f - \frac{i}{2} g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) + \\
& \quad \frac{i}{2} \left(\operatorname{PolyLog} [2, \left(\left(\frac{i}{2} c f + \sqrt{-c^2 f^2 - g^2} \right) \left(\frac{i}{2} c f + g - \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) / \left(g \left(\frac{i}{2} c f + g + \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} [\right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{1}{4} \left(\pi + 2 \operatorname{ArcSinh}[c x] \right) \right) \right] - \operatorname{PolyLog}[2, \left(\left(c f + \frac{1}{2} \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \left. \left(-c f + \frac{1}{2} g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot}\left[\frac{1}{4} \left(\pi + 2 \operatorname{ArcSinh}[c x] \right) \right] \right) \right) / \\
& \left. \left(g \left(\frac{1}{2} c f + g + \frac{1}{2} \sqrt{-c^2 f^2 - g^2} \operatorname{Cot}\left[\frac{1}{4} \left(\pi + 2 \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right] \Bigg) - \\
& \left. \left(18 c f g^2 \operatorname{ArcSinh}[c x] \operatorname{Sinh}[2 \operatorname{ArcSinh}[c x]] - 2 g^3 \operatorname{Sinh}[3 \operatorname{ArcSinh}[c x]] \right) \right)
\end{aligned}$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{f + g x} dx$$

Optimal (type 4, 1536 leaves, 37 steps):

$$\begin{aligned}
& \frac{a d^2 (c^2 f^2 + g^2)^2 \sqrt{d + c^2 d x^2}}{g^5} + \frac{2 b c d^2 x \sqrt{d + c^2 d x^2}}{15 g \sqrt{1 + c^2 x^2}} - \\
& \frac{b c d^2 (c^2 f^2 + g^2)^2 x \sqrt{d + c^2 d x^2}}{g^5 \sqrt{1 + c^2 x^2}} - \frac{b c d^2 (c^2 f^2 + 2 g^2) x \sqrt{d + c^2 d x^2}}{3 g^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{b c^3 d^2 f x^2 \sqrt{d + c^2 d x^2}}{16 g^2 \sqrt{1 + c^2 x^2}} + \frac{b c^3 d^2 f (c^2 f^2 + 2 g^2) x^2 \sqrt{d + c^2 d x^2}}{4 g^4 \sqrt{1 + c^2 x^2}} - \frac{b c^3 d^2 x^3 \sqrt{d + c^2 d x^2}}{45 g \sqrt{1 + c^2 x^2}} - \\
& \frac{b c^3 d^2 (c^2 f^2 + 2 g^2) x^3 \sqrt{d + c^2 d x^2}}{9 g^3 \sqrt{1 + c^2 x^2}} + \frac{b c^5 d^2 f x^4 \sqrt{d + c^2 d x^2}}{16 g^2 \sqrt{1 + c^2 x^2}} - \frac{b c^5 d^2 x^5 \sqrt{d + c^2 d x^2}}{25 g \sqrt{1 + c^2 x^2}} + \\
& \frac{b d^2 (c^2 f^2 + g^2)^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{g^5} - \frac{c^2 d^2 f x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{8 g^2} - \\
& \frac{c^2 d^2 f (c^2 f^2 + 2 g^2) x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{2 g^4} - \\
& \frac{c^4 d^2 f x^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{4 g^2} - \frac{d^2 (1 + c^2 x^2) \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 g} + \\
& \frac{d^2 (c^2 f^2 + 2 g^2) (1 + c^2 x^2) \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 g^3} + \\
& \frac{d^2 (1 + c^2 x^2)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{5 g} + \frac{c d^2 f \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{16 b g^2 \sqrt{1 + c^2 x^2}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{c d^2 f (c^2 f^2 + 2 g^2) \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{4 b g^4 \sqrt{1 + c^2 x^2}} - \\
& \frac{c d^2 (c^2 f^2 + g^2)^2 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b g^5 \sqrt{1 + c^2 x^2}} - \\
& \frac{d^2 (c^2 f^2 + g^2)^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c g^6 (f + g x) \sqrt{1 + c^2 x^2}} + \\
& \frac{d^2 (c^2 f^2 + g^2)^2 \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c g^4 (f + g x)} - \\
& \frac{a d^2 (c^2 f^2 + g^2)^{5/2} \sqrt{d + c^2 d x^2} \operatorname{ArcTanh}\left[\frac{g - c^2 f x}{\sqrt{c^2 f^2 + g^2} \sqrt{1 + c^2 x^2}}\right]}{g^6 \sqrt{1 + c^2 x^2}} + \\
& \frac{b d^2 (c^2 f^2 + g^2)^{5/2} \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{g^6 \sqrt{1 + c^2 x^2}} - \\
& \frac{b d^2 (c^2 f^2 + g^2)^{5/2} \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{g^6 \sqrt{1 + c^2 x^2}} + \\
& \frac{b d^2 (c^2 f^2 + g^2)^{5/2} \sqrt{d + c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{g^6 \sqrt{1 + c^2 x^2}} - \\
& \frac{b d^2 (c^2 f^2 + g^2)^{5/2} \sqrt{d + c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{g^6 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 9270 leaves) :

$$\begin{aligned}
& \sqrt{d (1 + c^2 x^2)} \left(\frac{a d^2 (15 c^4 f^4 + 35 c^2 f^2 g^2 + 23 g^4)}{15 g^5} - \frac{a c^2 d^2 f (4 c^2 f^2 + 9 g^2) x}{8 g^4} + \right. \\
& \left. \frac{a c^2 d^2 (5 c^2 f^2 + 11 g^2) x^2}{15 g^3} - \frac{a c^4 d^2 f x^3}{4 g^2} + \frac{a c^4 d^2 x^4}{5 g} \right) + \frac{a d^{5/2} (c^2 f^2 + g^2)^{5/2} \operatorname{Log}[f + g x]}{g^6} - \\
& \frac{a c d^{5/2} f (8 c^4 f^4 + 20 c^2 f^2 g^2 + 15 g^4) \operatorname{Log}[c d x + \sqrt{d} \sqrt{d (1 + c^2 x^2)}]}{8 g^6} - \frac{1}{g^6} \\
& a d^{5/2} (c^2 f^2 + g^2)^{5/2} \operatorname{Log}[d g - c^2 d f x + \sqrt{d} \sqrt{c^2 f^2 + g^2} \sqrt{d (1 + c^2 x^2)}] + \\
& b d^2 \left(-\frac{c x \sqrt{d (1 + c^2 x^2)}}{g \sqrt{1 + c^2 x^2}} + \frac{\sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]}{g} - \frac{c f \sqrt{d (1 + c^2 x^2)} \operatorname{ArcSinh}[c x]^2}{2 g^2 \sqrt{1 + c^2 x^2}} + \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{g^2 \sqrt{1+c^2 x^2}} (c^2 f^2 + g^2) \sqrt{d (1+c^2 x^2)} \left(-\frac{\pm \pi \operatorname{ArcTanh} \left[\frac{-g c f \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcSinh}[c x] \right]}{\sqrt{c^2 f^2 + g^2}} \right]}{\sqrt{c^2 f^2 + g^2}} - \right. \\
& \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \operatorname{ArcTanh} \left[\frac{(c f - \pm g) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& 2 \operatorname{ArcCos} \left[-\frac{\pm c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(-c f - \pm g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \\
& \left. \left. \operatorname{ArcCos} \left[-\frac{\pm c f}{g} \right] - 2 \pm \left(\operatorname{ArcTanh} \left[\frac{(c f - \pm g) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(-c f - \pm g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{e^{-\frac{1}{2} \pm \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right)} \sqrt{-c^2 f^2 - g^2}}{\sqrt{2} \sqrt{-\pm g} \sqrt{c f + c g x}} \right] + \left(\operatorname{ArcCos} \left[-\frac{\pm c f}{g} \right] + \right. \\
& 2 \pm \left(\operatorname{ArcTanh} \left[\frac{(c f - \pm g) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \left. \left. \operatorname{ArcTanh} \left[\frac{(-c f - \pm g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{e^{\frac{1}{2} \pm \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right)} \sqrt{-c^2 f^2 - g^2}}{\sqrt{2} \sqrt{-\pm g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{\pm c f}{g} \right] + \right. \\
& 2 \pm \operatorname{ArcTanh} \left[\frac{(-c f - \pm g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \left. \operatorname{Log} [1 - \right. \\
& \left. \left(\pm \left(c f - \pm \sqrt{-c^2 f^2 - g^2} \right) \left(c f - \pm g - \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right) / \\
& \left(g \left(c f - \pm g + \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right] \right) \right) + \\
& \left(-\operatorname{ArcCos} \left[-\frac{\pm c f}{g} \right] + 2 \pm \operatorname{ArcTanh} \left[\frac{(-c f - \pm g) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} [1 - \\
& \left(\pm \left(c f + \pm \sqrt{-c^2 f^2 - g^2} \right) \left(c f - \pm g - \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right] \right) \right) \right) / \\
& \left(g \left(c f - \pm g + \sqrt{-c^2 f^2 - g^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - \pm \operatorname{ArcSinh}[c x] \right) \right] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{i}{2} \left(\operatorname{PolyLog}[2, \left(i \left(c f - i \sqrt{-c^2 f^2 - g^2} \right) \right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]\right) \right) \Big/ \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]\right) \right) - \operatorname{PolyLog}[2, \left(i \left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \quad \left(c f - i g - \sqrt{-c^2 f^2 - g^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]\right) \Big) \Big/ \\
& \quad \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]\right) \right) \right) \Bigg) + \\
2 b d^2 & \left(\frac{1}{8 \sqrt{1+c^2 x^2}} \sqrt{d (1+c^2 x^2)} \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-g+c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2+g^2}}\right]}{\sqrt{c^2 f^2+g^2}} + \right. \right. \\
& \quad \frac{1}{\sqrt{-c^2 f^2-g^2}} \left(2 \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(c f+i g) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right. \\
& \quad (\pi-2 i \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Tan}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \\
& \quad \left. \left. \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f+i g) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \\
& \quad 2 i \operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Tan}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \\
& \quad \operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2-g^2}}{\sqrt{-i g} \sqrt{c f+c g x}}\right] + \\
& \quad \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f+i g) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right. \right. \\
& \quad \left. \left. \operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Tan}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right]\right) \right) \\
& \quad \operatorname{Log}\left[\frac{\left(\frac{1}{2}+\frac{i}{2}\right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2-g^2}}{\sqrt{-i g} \sqrt{c f+c g x}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\pi - 2 \operatorname{ArcSinh}[c x] \right) \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \\
& \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] - 2 \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} \left[\right. \\
& \left. \left(\frac{1}{2} - \frac{i}{2} \right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \frac{\sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] + \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2} \right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} \left[\right. \\
& \left. \left((i c f + g) \left(-i c f + g + \sqrt{-c^2 f^2 - g^2} \right) \left(1 + i \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] / \\
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] - \\
& \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \\
& \operatorname{Log} \left[\left((i c f + g) \left(i c f - g + \sqrt{-c^2 f^2 - g^2} \right) \left(i + \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] / \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] + \\
& i \left(\operatorname{PolyLog} [2, \left(\left(i c f + \sqrt{-c^2 f^2 - g^2} \right) \left(i c f + g - i \sqrt{-c^2 f^2 - g^2} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) / \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} [\right. \right. \right. \\
& \left. \left. \left. \frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right)] - \operatorname{PolyLog} [2, \left(\left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \left. \left. \left(-c f + i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] \Bigg) - \\
& \left. \left(18 c f g^2 \operatorname{ArcSinh}[c x] \operatorname{Sinh}[2 \operatorname{ArcSinh}[c x]] - 2 g^3 \operatorname{Sinh}[3 \operatorname{ArcSinh}[c x]] \right) \right] \Bigg) + \\
& b d^2 \left(- \frac{1}{32 g^2 \sqrt{1+c^2 x^2}} \sqrt{d (1+c^2 x^2)} \left(-2 c g x + 2 g \sqrt{1+c^2 x^2} \operatorname{ArcSinh}[c x] - \right. \right. \\
& \left. \left. c f \operatorname{ArcSinh}[c x]^2 + \right. \right. \\
& \left. \left. (2 c^2 f^2 + g^2) \left(- \frac{i \pi \operatorname{ArcTanh} \left[\frac{-g+c f \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcSinh}[c x] \right]}{\sqrt{c^2 f^2 + g^2}} \right]}{\sqrt{c^2 f^2 + g^2}} - \frac{1}{\sqrt{-c^2 f^2 - g^2}} \right. \right. \right. \\
& \left. \left. \left. 2 \operatorname{ArcCos} \left[- \frac{i c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \left. \left. \left. (\pi - 2 i \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \left. \left. \left. \operatorname{ArcCos} \left[- \frac{i c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \right. \\
& \left. \left. \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right] \right) \right. \\
& \left. \left. \left. \operatorname{Log} \left[\frac{\left(\frac{1}{2} - \frac{i}{2} \right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] + \left(\operatorname{ArcCos} \left[- \frac{i c f}{g} \right] + \right. \right. \right. \\
& \left. \left. \left. 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2} \right) e^{\frac{1}{2} \text{ArcSinh}[cx]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] - \left(\text{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& \left. 2 i \text{ArcTanh} \left[\frac{(c f + i g) \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \text{Log} \left[\right. \\
& \left. \left((i c f + g) \left(-i c f + g + \sqrt{-c^2 f^2 - g^2} \right) \left(1 + i \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right] / \\
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right] - \\
& \left. \left(\text{ArcCos} \left[-\frac{i c f}{g} \right] - 2 i \text{ArcTanh} \left[\frac{(c f + i g) \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \text{Log} \left[\right. \right. \\
& \left. \left. \left((i c f + g) \left(i c f - g + \sqrt{-c^2 f^2 - g^2} \right) \left(i + \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right] / \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right] + i \\
& \left. \left(\text{PolyLog} \left[2, \left(\left(i c f + \sqrt{-c^2 f^2 - g^2} \right) \left(i c f + g - i \sqrt{-c^2 f^2 - g^2} \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right) / \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \text{Cot} \left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right] - \text{PolyLog} \left[2, \left(\left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \right. \right. \\
& \left. \left. \left(-c f + i g + \sqrt{-c^2 f^2 - g^2} \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right) / \\
& \left. \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right] \right) \right) \right) \right] \right) \right] + \\
& \frac{1}{16 \sqrt{1 + c^2 x^2}} \sqrt{d (1 + c^2 x^2)} \left(-\frac{\frac{i \pi \text{ArcTanh} \left[\frac{-g+c f \tanh \left[\frac{1}{2} \text{ArcSinh}[cx] \right]}{\sqrt{c^2 f^2 + g^2}} \right]}{\sqrt{c^2 f^2 + g^2}} - \right. \\
& \left. \frac{1}{\sqrt{-c^2 f^2 - g^2}} \right. \\
& \left. \left(2 \text{ArcCos} \left[-\frac{i c f}{g} \right] \text{ArcTanh} \left[\frac{(c f + i g) \text{Cot} \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \left. \left. (\pi - 2 i \text{ArcSinh}[cx]) \text{ArcTanh} \left[\frac{(c f - i g) \tan \left[\frac{1}{4} (\pi + 2 i \text{ArcSinh}[cx]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] - 2 \operatorname{i} \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \quad \left. 2 \operatorname{i} \operatorname{ArcTanh} \left[\frac{(c f - \frac{i}{2} g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \\
& \operatorname{Log} \left[\frac{\left(\frac{1}{2} - \frac{i}{2}\right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-\frac{i}{2} g} \sqrt{c f + c g x}} \right] + \\
& \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] + 2 \operatorname{i} \left(\operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \quad \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - \frac{i}{2} g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-\frac{i}{2} g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] + \right. \\
& \quad \left. 2 \operatorname{i} \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \\
& \operatorname{Log} \left[\left((\frac{i}{2} c f + g) \left(-\frac{i}{2} c f + g + \sqrt{-c^2 f^2 - g^2} \right) \left(1 + \operatorname{i} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] / \\
& \quad \left(g \left(\frac{i}{2} c f + g + \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right) - \\
& \left(\operatorname{ArcCos} \left[-\frac{\frac{i}{2} c f}{g} \right] - 2 \operatorname{i} \operatorname{ArcTanh} \left[\frac{(c f + \frac{i}{2} g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \\
& \operatorname{Log} \left[\left((\frac{i}{2} c f + g) \left(\frac{i}{2} c f - g + \sqrt{-c^2 f^2 - g^2} \right) \left(\frac{i}{2} + \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] / \\
& \quad \left(g \left(c f - \frac{i}{2} g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right) + \\
& \operatorname{i} \left(\operatorname{PolyLog} [2, \left(\left(\frac{i}{2} c f + \sqrt{-c^2 f^2 - g^2} \right) \left(\frac{i}{2} c f + g - \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right) / \left(g \left(\frac{i}{2} c f + g + \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \quad \left. \left. \left. \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right)] - \operatorname{PolyLog} [2, \left(\left(c f + \frac{i}{2} \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \quad \left. \left. \left(-c f + \frac{i}{2} g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 \operatorname{i} \operatorname{ArcSinh}[c x]) \right] \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right) - \\
& \frac{1}{144 g^4 \sqrt{1 + c^2 x^2}} \sqrt{d (1 + c^2 x^2)} \left(-18 c g (4 c^2 f^2 + g^2) x + 18 g (4 c^2 f^2 + g^2) \right. \\
& \left. \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x] - \right. \\
& 18 c f (2 c^2 f^2 + g^2) \operatorname{ArcSinh}[c x]^2 + 9 c f g^2 \\
& \operatorname{Cosh}[2 \operatorname{ArcSinh}[c x]] + \\
& 6 g^3 \operatorname{ArcSinh}[c x] \operatorname{Cosh}[3 \operatorname{ArcSinh}[c x]] + \\
& 9 (8 c^4 f^4 + 8 c^2 f^2 g^2 + g^4) \\
& - \frac{i \pi \operatorname{ArcTanh} \left[\frac{-g + c f \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcSinh}[c x] \right]}{\sqrt{c^2 f^2 + g^2}} \right]}{\sqrt{c^2 f^2 + g^2}} - \\
& \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \operatorname{ArcCos} \left[-\frac{i c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \\
& (\pi - 2 i \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \\
& \left. \operatorname{ArcCos} \left[-\frac{i c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] - \right. \\
& \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \operatorname{Log} \left[\right. \\
& \left. \left(\frac{1}{2} - \frac{i}{2} \right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2} \right] + \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \\
& \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2} \right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(c f + i g \right) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \operatorname{Log} \left[\right. \\
& \left. \left((i c f + g) \left(-i c f + g + \sqrt{-c^2 f^2 - g^2} \right) \left(1 + i \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right. \\
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] - \\
& \left. \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right] \\
& \operatorname{Log} \left[\left((i c f + g) \left(i c f - g + \sqrt{-c^2 f^2 - g^2} \right) \left(i + \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right. \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] + \\
& i \left(\operatorname{PolyLog} [2, \left(\left(i c f + \sqrt{-c^2 f^2 - g^2} \right) \left(i c f + g - i \sqrt{-c^2 f^2 - g^2} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right) \Big/ \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\right. \right. \right. \\
& \left. \left. \left. \frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \Big) - \operatorname{PolyLog} [2, \left(\left(c f + i \sqrt{-c^2 f^2 - g^2} \right) \right. \\
& \left. \left(-c f + i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \Big) \Big/ \\
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right] \Big) \Big) - \\
& \left. \left. \left. \left. 18 c f g^2 \operatorname{ArcSinh}[c x] \operatorname{Sinh}[2 \operatorname{ArcSinh}[c x]] - 2 g^3 \operatorname{Sinh}[3 \operatorname{ArcSinh}[c x]] \right) \right] \right) + \\
& \frac{1}{32 \sqrt{1 + c^2 x^2}} \sqrt{d (1 + c^2 x^2)} \\
& \left(-\frac{32 c^5 f^4 x}{g^5} - \right. \\
& \left. \frac{24 c^3 f^2 x}{g^3} - \right. \\
& \left. \frac{2 c x}{g} + \right. \\
& \left. \frac{2 (16 c^4 f^4 + 12 c^2 f^2 g^2 + g^4) \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{g^5} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{16 c^5 f^5 \operatorname{ArcSinh}[c x]^2}{g^6} - \\
& \frac{16 c^3 f^3 \operatorname{ArcSinh}[c x]^2}{g^4} - \\
& \frac{3 c f \operatorname{ArcSinh}[c x]^2}{g^2} + \\
& \frac{2 c f (2 c^2 f^2 + g^2) \operatorname{Cosh}[2 \operatorname{ArcSinh}[c x]]}{g^4} + \\
& \frac{8 c^2 f^2 \operatorname{ArcSinh}[c x] \operatorname{Cosh}[3 \operatorname{ArcSinh}[c x]]}{3 g^3} + \\
& \frac{2 \operatorname{ArcSinh}[c x] \operatorname{Cosh}[3 \operatorname{ArcSinh}[c x]]}{3 g} + \\
& \frac{c f \operatorname{Cosh}[4 \operatorname{ArcSinh}[c x]]}{4 g^2} + \\
& \frac{2 \operatorname{ArcSinh}[c x] \operatorname{Cosh}[5 \operatorname{ArcSinh}[c x]]}{5 g} + \\
& \frac{1}{g^6} (2 c^2 f^2 + g^2) (16 c^4 f^4 + 16 c^2 f^2 g^2 + g^4) \\
& \left(- \frac{\frac{i \pi \operatorname{ArcTanh}\left[\frac{-g+c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2+g^2}}\right]}{\sqrt{c^2 f^2+g^2}} - \frac{1}{\sqrt{-c^2 f^2-g^2}} \right. \\
& \left. \left(2 \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(c f+i g) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right. \right. \\
& \left. \left. (\pi-2 i \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Tan}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right. \\
& \left. \left. \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f+i g) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] - \right. \right. \\
& \left. \left. 2 i \operatorname{ArcTanh}\left[\frac{(c f-i g) \operatorname{Tan}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] \right) \\
& \operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2-g^2}}{\sqrt{-i g} \sqrt{c f+c g x}}\right] + \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \left. \left. 2 i \operatorname{ArcTanh}\left[\frac{(c f+i g) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2-g^2}}\right] + \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{ArcTanh}\left[\frac{\left(c f - i g\right) \tan\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right] \Bigg) \\
& \operatorname{Log}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(c f + i g) \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right]\right) \operatorname{Log}\left[\right. \\
& \left. \left(i c f + g\right) \left(-i c f + g + \sqrt{-c^2 f^2 - g^2}\right) \left(1 + i \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right)\right] / \\
& \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2}\right) \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right] - \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + i g) \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right]\right) \operatorname{Log}\left[\right. \right. \\
& \left. \left(i c f + g\right) \left(i c f - g + \sqrt{-c^2 f^2 - g^2}\right) \left(i + \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right)\right] / \\
& \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2}\right) \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right] + i \\
& \left. \left(\operatorname{PolyLog}\left[2, \left(i c f + \sqrt{-c^2 f^2 - g^2}\right) \left(i c f + g - i \sqrt{-c^2 f^2 - g^2}\right.\right.\right.\right. \\
& \left.\left.\left.\left.\cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right)\right]\right) / \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2}\right) \cot\left[\right. \right. \\
& \left.\left.\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right) \left.\right] - \operatorname{PolyLog}\left[2, \left(\left(c f + i \sqrt{-c^2 f^2 - g^2}\right)\right.\right. \\
& \left.\left.- c f + i g + \sqrt{-c^2 f^2 - g^2}\right) \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right)\right] / \\
& \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2}\right) \cot\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]\right)\right] \Bigg) - \\
& \frac{8 c^3 f^3 \operatorname{ArcSinh}[c x] \sinh[2 \operatorname{ArcSinh}[c x]]}{g^4} - \frac{4 c f \operatorname{ArcSinh}[c x] \sinh[2 \operatorname{ArcSinh}[c x]]}{g^2} - \\
& \frac{8 c^2 f^2 \sinh[3 \operatorname{ArcSinh}[c x]]}{9 g^3} - \\
& \frac{2 \sinh[3 \operatorname{ArcSinh}[c x]]}{9 g} - \\
& \frac{c f \operatorname{ArcSinh}[c x] \sinh[4 \operatorname{ArcSinh}[c x]]}{g^2}
\end{aligned}$$

$$\left. \frac{2 \operatorname{Sinh}[5 \operatorname{ArcSinh}[c x]]}{25 g} \right)$$

Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{(f + g x) \sqrt{d + c^2 d x^2}} dx$$

Optimal (type 4, 325 leaves, 10 steps):

$$\begin{aligned} & \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2}} - \\ & \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2}} + \\ & \frac{b \sqrt{1 + c^2 x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right] - b \sqrt{1 + c^2 x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2} \sqrt{d + c^2 d x^2}} \end{aligned}$$

Result (type 4, 1233 leaves):

$$\begin{aligned} & \frac{a \operatorname{Log}[f + g x]}{\sqrt{d} \sqrt{c^2 f^2 + g^2}} - \frac{a \operatorname{Log}[d g - c^2 d f x + \sqrt{d} \sqrt{c^2 f^2 + g^2} \sqrt{d (1 + c^2 x^2)}]}{\sqrt{d} \sqrt{c^2 f^2 + g^2}} + \\ & \frac{1}{\sqrt{d (1 + c^2 x^2)}} b \sqrt{1 + c^2 x^2} \left(-\frac{\frac{i \pi \operatorname{ArcTanh}\left[\frac{-g + c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2}} - \right. \\ & \left. \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(c f + i g) \operatorname{Cot}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right] + \right. \right. \\ & \left. \left. (\pi - 2 i \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}\left[\frac{(c f - i g) \operatorname{Tan}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right] + \right. \right. \\ & \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + i g) \operatorname{Cot}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{-c^2 f^2 - g^2}}\right] \right) - \right. \right) \end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \\
& \operatorname{Log} \left[\frac{\left(\frac{1}{2} - \frac{i}{2} \right) e^{-\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] + \\
& \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] + \right. \right. \\
& \quad \left. \left. \operatorname{ArcTanh} \left[\frac{(c f - i g) \operatorname{Tan} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right] \\
& \operatorname{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2} \right) e^{\frac{1}{2} \operatorname{ArcSinh}[c x]} \sqrt{-c^2 f^2 - g^2}}{\sqrt{-i g} \sqrt{c f + c g x}} \right] - \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] + \right. \\
& \quad \left. \left. 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right] \\
& \operatorname{Log} \left[\left((i c f + g) \left(-i c f + g + \sqrt{-c^2 f^2 - g^2} \right) \left(1 + i \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) / \right. \\
& \quad \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) - \right. \\
& \quad \left. \left(\operatorname{ArcCos} \left[-\frac{i c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + i g) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right]}{\sqrt{-c^2 f^2 - g^2}} \right] \right) \right] \\
& \operatorname{Log} \left[\left((i c f + g) \left(i c f - g + \sqrt{-c^2 f^2 - g^2} \right) \left(i + \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) / \right. \\
& \quad \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) + i \left(\operatorname{PolyLog} [2, \right. \right. \\
& \quad \left. \left. \left((i c f + \sqrt{-c^2 f^2 - g^2}) \left(i c f + g - i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right) / \right. \\
& \quad \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right)] - \operatorname{PolyLog} [2, \right. \\
& \quad \left. \left. \left((c f + i \sqrt{-c^2 f^2 - g^2}) \left(-c f + i g + \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right) / \right. \\
& \quad \left. \left. \left(g \left(i c f + g + i \sqrt{-c^2 f^2 - g^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x]) \right] \right) \right) \right) \right]
\end{aligned}$$

Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{(f + g x)^2 \sqrt{d + c^2 d x^2}} dx$$

Optimal (type 4, 444 leaves, 13 steps):

$$\begin{aligned} & -\frac{g (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])}{(c^2 f^2 + g^2) (f + g x) \sqrt{d + c^2 d x^2}} + \frac{c^2 f \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{(c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2}} - \\ & \frac{c^2 f \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{(c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2}} + \frac{b c \sqrt{1 + c^2 x^2} \operatorname{Log}[f + g x]}{(c^2 f^2 + g^2) \sqrt{d + c^2 d x^2}} + \\ & \frac{b c^2 f \sqrt{1 + c^2 x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{(c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2}} - \frac{b c^2 f \sqrt{1 + c^2 x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{(c^2 f^2 + g^2)^{3/2} \sqrt{d + c^2 d x^2}} \end{aligned}$$

Result (type 4, 1586 leaves):

$$\begin{aligned} & -\frac{a g \sqrt{d (1 + c^2 x^2)}}{d (c^2 f^2 + g^2) (f + g x)} + \frac{a c^2 f \operatorname{Log}[f + g x]}{\sqrt{d} (c f - i g) (c f + i g) \sqrt{c^2 f^2 + g^2}} - \\ & \frac{a c^2 f \operatorname{Log}[d g - c^2 d f x + \sqrt{d} \sqrt{c^2 f^2 + g^2} \sqrt{d (1 + c^2 x^2)}]}{\sqrt{d} (c f - i g) (c f + i g) \sqrt{c^2 f^2 + g^2}} + \\ & b c \left(-\frac{g (1 + c^2 x^2) \operatorname{ArcSinh}[c x]}{(c^2 f^2 + g^2) (c f + c g x) \sqrt{d (1 + c^2 x^2)}} + \frac{\sqrt{1 + c^2 x^2} \operatorname{Log}\left[1 + \frac{g x}{f}\right]}{(c^2 f^2 + g^2) \sqrt{d (1 + c^2 x^2)}} + \right. \\ & \left. \frac{1}{(c^2 f^2 + g^2) \sqrt{d (1 + c^2 x^2)}} c f \sqrt{1 + c^2 x^2} \left(-\frac{i \pi \operatorname{ArcTanh}\left[\frac{-g + c f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c x]\right]}{\sqrt{c^2 f^2 + g^2}}\right]}{\sqrt{c^2 f^2 + g^2}} - \right. \right. \\ & \left. \left. \frac{1}{\sqrt{-c^2 f^2 - g^2}} \left(2 \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x] \right) \operatorname{ArcTanh}\left[\frac{(c f - i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right] - \right. \right. \\ & \left. \left. 2 \operatorname{ArcCos}\left[-\frac{i c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right] + \right. \right. \\ & \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i c f}{g}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f - i g) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right] - \right. \right. \right. \right. \\ & \left. \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f - i g) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right]\right)\right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{Log}\left[\frac{e^{-\frac{1}{2} i \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)} \sqrt{-c^2 f^2 - g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f + c g x}}\right] + \left(\text{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& 2 i \left(\text{ArcTanh}\left[\frac{(c f - i g) \text{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right] - \right. \\
& \left. \left. \text{ArcTanh}\left[\frac{(-c f - i g) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right]\right) \right) \\
& \text{Log}\left[\frac{e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)} \sqrt{-c^2 f^2 - g^2}}{\sqrt{2} \sqrt{-i g} \sqrt{c f + c g x}}\right] - \left(\text{ArcCos}\left[-\frac{i c f}{g}\right] + \right. \\
& 2 i \text{ArcTanh}\left[\frac{(-c f - i g) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right] \left. \text{Log}[1 - \right. \\
& \left. \left(i \left(c f - i \sqrt{-c^2 f^2 - g^2}\right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2}\right) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right) / \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2}\right) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right] + \\
& \left(-\text{ArcCos}\left[-\frac{i c f}{g}\right] + 2 i \text{ArcTanh}\left[\frac{(-c f - i g) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]}{\sqrt{-c^2 f^2 - g^2}}\right] \right) \text{Log}[1 - \\
& \left. \left(i \left(c f + i \sqrt{-c^2 f^2 - g^2}\right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2}\right) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right) / \\
& \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2}\right) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right] + \\
& i \left(\text{PolyLog}\left[2, \left(i \left(c f - i \sqrt{-c^2 f^2 - g^2}\right) \left(c f - i g - \sqrt{-c^2 f^2 - g^2}\right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right)\right) / \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2}\right) \right. \\
& \left. \left. \left. \left. \left. \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right)\right] - \text{PolyLog}\left[2, \left(i \left(c f + i \sqrt{-c^2 f^2 - g^2}\right) \right. \right. \\
& \left. \left. \left. \left. \left. \left(c f - i g - \sqrt{-c^2 f^2 - g^2}\right) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right)\right) / \right. \\
& \left. \left. \left. \left. \left. \left(g \left(c f - i g + \sqrt{-c^2 f^2 - g^2}\right) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[c x]\right)\right]\right)\right)\right)\right] \right)
\end{aligned}$$

Problem 54: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \text{ArcSinh}[c x])^2 \text{Log}[h (f + g x)^m]}{\sqrt{1 + c^2 x^2}} dx$$

Optimal (type 4, 438 leaves, 13 steps):

$$\begin{aligned}
 & \frac{m (a + b \operatorname{ArcSinh}[c x])^4}{12 b^2 c} - \frac{m (a + b \operatorname{ArcSinh}[c x])^3 \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{3 b c} - \\
 & \frac{m (a + b \operatorname{ArcSinh}[c x])^3 \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{3 b c} + \frac{(a + b \operatorname{ArcSinh}[c x])^3 \operatorname{Log}[h (f + g x)^m]}{3 b c} - \\
 & \frac{m (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{PolyLog}[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}]}{c} - \\
 & \frac{m (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{PolyLog}[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}]}{c} + \\
 & \frac{2 b m (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[3, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}]}{c} + \\
 & \frac{2 b m (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[3, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}]}{c} - \\
 & \frac{2 b^2 m \operatorname{PolyLog}[4, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}]}{c} - \frac{2 b^2 m \operatorname{PolyLog}[4, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}]}{c}
 \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 55: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[h (f + g x)^m]}{\sqrt{1 + c^2 x^2}} dx$$

Optimal (type 4, 332 leaves, 11 steps):

$$\begin{aligned}
& \frac{m (a + b \operatorname{ArcSinh}[c x])^3}{6 b^2 c} - \frac{m (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{2 b c} - \\
& \frac{m (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{2 b c} + \frac{(a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[h (f + g x)^m]}{2 b c} - \\
& \frac{m (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}]}{c} - \\
& \frac{m (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}]}{c} + \\
& \frac{b m \operatorname{PolyLog}[3, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}]}{c} + \frac{b m \operatorname{PolyLog}[3, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}]}{c}
\end{aligned}$$

Result (type 4, 1547 leaves):

$$\begin{aligned}
& -\frac{1}{24 c} \left(3 a m \pi^2 - 12 i a m \pi \operatorname{ArcSinh}[c x] - 12 a m \operatorname{ArcSinh}[c x]^2 - 4 b m \operatorname{ArcSinh}[c x]^3 - \right. \\
& 96 a m \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c f}{g}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(c f + i g) \operatorname{Cot}\left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[c x])\right]}{\sqrt{c^2 f^2 + g^2}}\right] + \\
& 12 b m \operatorname{ArcSinh}[c x]^2 \operatorname{Log}\left[\frac{-c f - e^{\operatorname{ArcSinh}[c x]} g + \sqrt{c^2 f^2 + g^2}}{-c f + \sqrt{c^2 f^2 + g^2}}\right] + \\
& 12 b m \operatorname{ArcSinh}[c x]^2 \operatorname{Log}\left[\frac{c f + e^{\operatorname{ArcSinh}[c x]} g + \sqrt{c^2 f^2 + g^2}}{c f + \sqrt{c^2 f^2 + g^2}}\right] + \\
& 12 i a m \pi \operatorname{Log}\left[\frac{-c e^{\operatorname{ArcSinh}[c x]} f + g - e^{\operatorname{ArcSinh}[c x]} \sqrt{c^2 f^2 + g^2}}{g}\right] - \\
& 48 i a m \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[\frac{-c e^{\operatorname{ArcSinh}[c x]} f + g - e^{\operatorname{ArcSinh}[c x]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 24 a m \operatorname{ArcSinh}[c x] \operatorname{Log}\left[\frac{-c e^{\operatorname{ArcSinh}[c x]} f + g - e^{\operatorname{ArcSinh}[c x]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& \left. 12 i b m \pi \operatorname{ArcSinh}[c x] \operatorname{Log}\left[\frac{-c e^{\operatorname{ArcSinh}[c x]} f + g - e^{\operatorname{ArcSinh}[c x]} \sqrt{c^2 f^2 + g^2}}{g}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& 48 \text{ } \text{im} \text{ ArcSin}\left[\frac{\sqrt{1 + \frac{icf}{g}}}{\sqrt{2}}\right] \text{ ArcSinh}[cx] \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g - e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 12 \text{ } \text{bm} \text{ ArcSinh}[cx]^2 \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g - e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 12 \text{ } \text{am} \pi \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g + e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 48 \text{ } \text{am} \text{ ArcSin}\left[\frac{\sqrt{1 + \frac{icf}{g}}}{\sqrt{2}}\right] \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g + e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 24 \text{ } \text{am} \text{ ArcSinh}[cx] \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g + e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 12 \text{ } \text{bm} \pi \text{ ArcSinh}[cx] \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g + e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 48 \text{ } \text{bm} \text{ ArcSin}\left[\frac{\sqrt{1 + \frac{icf}{g}}}{\sqrt{2}}\right] \text{ ArcSinh}[cx] \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g + e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] + \\
& 12 \text{ } \text{bm} \text{ ArcSinh}[cx]^2 \text{ Log}\left[\frac{-c e^{\text{ArcSinh}[cx]} f + g + e^{\text{ArcSinh}[cx]} \sqrt{c^2 f^2 + g^2}}{g}\right] - \\
& 12 \text{ } \text{am} \pi \text{ Log}[c (f + g x)] - 24 \text{ } \text{am} \text{ ArcSinh}[cx] \text{ Log}[h (f + g x)^m] - 12 \text{ } \text{bm} \text{ ArcSinh}[cx]^2 \\
& \text{Log}[h (f + g x)^m] - 12 \text{ } \text{bm} \pi \text{ ArcSinh}[cx] \text{ Log}\left[1 + \frac{(-c f + \sqrt{c^2 f^2 + g^2}) (c x + \sqrt{1 + c^2 x^2})}{g}\right] - \\
& 48 \text{ } \text{bm} \text{ ArcSin}\left[\frac{\sqrt{1 + \frac{icf}{g}}}{\sqrt{2}}\right] \text{ ArcSinh}[cx] \text{ Log}\left[1 + \frac{(-c f + \sqrt{c^2 f^2 + g^2}) (c x + \sqrt{1 + c^2 x^2})}{g}\right] - \\
& 12 \text{ } \text{bm} \text{ ArcSinh}[cx]^2 \text{ Log}\left[1 + \frac{(-c f + \sqrt{c^2 f^2 + g^2}) (c x + \sqrt{1 + c^2 x^2})}{g}\right] - \\
& 12 \text{ } \text{bm} \pi \text{ ArcSinh}[cx] \text{ Log}\left[1 - \frac{(c f + \sqrt{c^2 f^2 + g^2}) (c x + \sqrt{1 + c^2 x^2})}{g}\right] + \\
& 48 \text{ } \text{bm} \text{ ArcSin}\left[\frac{\sqrt{1 + \frac{icf}{g}}}{\sqrt{2}}\right] \text{ ArcSinh}[cx] \text{ Log}\left[1 - \frac{(c f + \sqrt{c^2 f^2 + g^2}) (c x + \sqrt{1 + c^2 x^2})}{g}\right] -
\end{aligned}$$

$$\begin{aligned}
& 12 b m \operatorname{ArcSinh}[c x]^2 \operatorname{Log}\left[1 - \frac{\left(c f + \sqrt{c^2 f^2 + g^2}\right) \left(c x + \sqrt{1 + c^2 x^2}\right)}{g}\right] + \\
& 24 a m \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[c x]} \left(c f - \sqrt{c^2 f^2 + g^2}\right)}{g}\right] + \\
& 24 b m \operatorname{ArcSinh}[c x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[c x]} g}{-c f + \sqrt{c^2 f^2 + g^2}}\right] + 24 b m \operatorname{ArcSinh}[c x] \\
& \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right] + 24 a m \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[c x]} \left(c f + \sqrt{c^2 f^2 + g^2}\right)}{g}\right] - \\
& 24 b m \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[c x]} g}{-c f + \sqrt{c^2 f^2 + g^2}}\right] - 24 b m \operatorname{PolyLog}\left[3, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]
\end{aligned}$$

Problem 56: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Log}\left[h (f + g x)^m\right]}{\sqrt{1 + c^2 x^2}} dx$$

Optimal (type 4, 197 leaves, 9 steps) :

$$\begin{aligned}
& \frac{m \operatorname{ArcSinh}[c x]^2}{c} - \frac{m \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{c} - \frac{m \operatorname{ArcSinh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{c} + \\
& \frac{\operatorname{ArcSinh}[c x] \operatorname{Log}\left[h (f + g x)^m\right]}{c} - \frac{m \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f - \sqrt{c^2 f^2 + g^2}}\right]}{c} - \frac{m \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcSinh}[c x]} g}{c f + \sqrt{c^2 f^2 + g^2}}\right]}{c}
\end{aligned}$$

Result (type 1, 1 leaves) :

???

Problem 62: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSinh}[a + b x]}{x} dx$$

Optimal (type 4, 131 leaves, 9 steps) :

$$\begin{aligned}
& -\frac{1}{2} \operatorname{ArcSinh}[a+b x]^2 + \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + \\
& \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] + \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]
\end{aligned}$$

Result (type 4, 290 leaves):

$$\begin{aligned}
& \frac{1}{8} \left((\pi - 2 \operatorname{ArcSinh}[a+b x])^2 + \right. \\
& 32 \operatorname{ArcSin}\left[\frac{\sqrt{1-i a}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(-i+a) \operatorname{Cot}\left[\frac{1}{4}(\pi+2 \operatorname{ArcSinh}[a+b x])\right]}{\sqrt{1+a^2}}\right] + \\
& 4 i \left(\pi - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1-i a}}{\sqrt{2}}\right] - 2 i \operatorname{ArcSinh}[a+b x] \right) \\
& \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]} - \sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] + 4 i \\
& \left(\pi + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1-i a}}{\sqrt{2}}\right] - 2 i \operatorname{ArcSinh}[a+b x] \right) \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]} + \sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] + \\
& 8 \operatorname{ArcSinh}[a+b x] \operatorname{Log}[b x] - 4 (\pi + 2 \operatorname{ArcSinh}[a+b x]) \operatorname{Log}[b x] + \\
& \left. 8 \operatorname{PolyLog}\left[2, \left(-a+\sqrt{1+a^2}\right) e^{\operatorname{ArcSinh}[a+b x]}\right] + 8 \operatorname{PolyLog}\left[2, -\left(a+\sqrt{1+a^2}\right) e^{\operatorname{ArcSinh}[a+b x]}\right] \right)
\end{aligned}$$

Problem 71: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSinh}[a+b x]^2}{x} dx$$

Optimal (type 4, 205 leaves, 11 steps):

$$\begin{aligned}
& -\frac{1}{3} \operatorname{ArcSinh}[a+b x]^3 + \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + \\
& \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] + 2 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + \\
& 2 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] - \\
& 2 \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] - 2 \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]
\end{aligned}$$

Result (type 4, 890 leaves):

$$\begin{aligned}
& -\frac{1}{3} \operatorname{ArcSinh}[a+b x]^3 + \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[\frac{a+\sqrt{1+a^2}-e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] + \\
& \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[\frac{-a+\sqrt{1+a^2}+e^{\operatorname{ArcSinh}[a+b x]}}{-a+\sqrt{1+a^2}}\right] + \\
& \pm \pi \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]}-\sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] - \\
& 4 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1-\pm a}}{\sqrt{2}}\right] \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]}-\sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] + \\
& \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]}-\sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] + \\
& \pm \pi \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]}+\sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] + \\
& 4 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1-\pm a}}{\sqrt{2}}\right] \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]}+\sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] + \\
& \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1+a e^{\operatorname{ArcSinh}[a+b x]}+\sqrt{1+a^2} e^{\operatorname{ArcSinh}[a+b x]}\right] - \\
& \pm \pi \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1+\left(a-\sqrt{1+a^2}\right)(a+b x)+\left(a-\sqrt{1+a^2}\right) \sqrt{1+(a+b x)^2}\right] + \\
& 4 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1-\pm a}}{\sqrt{2}}\right] \operatorname{ArcSinh}[a+b x] \\
& \operatorname{Log}\left[1+\left(a-\sqrt{1+a^2}\right)(a+b x)+\left(a-\sqrt{1+a^2}\right) \sqrt{1+(a+b x)^2}\right] - \\
& \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1+\left(a-\sqrt{1+a^2}\right)(a+b x)+\left(a-\sqrt{1+a^2}\right) \sqrt{1+(a+b x)^2}\right] - \\
& \pm \pi \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1+\left(a+\sqrt{1+a^2}\right)(a+b x)+\left(a+\sqrt{1+a^2}\right) \sqrt{1+(a+b x)^2}\right] - \\
& 4 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1-\pm a}}{\sqrt{2}}\right] \operatorname{ArcSinh}[a+b x] \\
& \operatorname{Log}\left[1+\left(a+\sqrt{1+a^2}\right)(a+b x)+\left(a+\sqrt{1+a^2}\right) \sqrt{1+(a+b x)^2}\right] - \\
& \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1+\left(a+\sqrt{1+a^2}\right)(a+b x)+\left(a+\sqrt{1+a^2}\right) \sqrt{1+(a+b x)^2}\right] + \\
& 2 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + 2 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] - \\
& 2 \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] - 2 \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]
\end{aligned}$$

Problem 72: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSinh}[a+b x]^2}{x^2} dx$$

Optimal (type 4, 178 leaves, 11 steps):

$$\begin{aligned}
 & -\frac{\text{ArcSinh}[a+b x]^2}{x} - \frac{2 b \text{ArcSinh}[a+b x] \log \left[1-\frac{e^{\text{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} + \\
 & \frac{2 b \text{ArcSinh}[a+b x] \log \left[1-\frac{e^{\text{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} - \frac{2 b \text{PolyLog}\left[2, \frac{e^{\text{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} + \frac{2 b \text{PolyLog}\left[2, \frac{e^{\text{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}}
 \end{aligned}$$

Result (type 4, 866 leaves) :

$$\begin{aligned}
& - \frac{\text{ArcSinh}[a + b x]^2}{x} - \frac{2 \pm b \pi \text{ArcTanh}\left[\frac{-1-a \tanh\left[\frac{1}{2} \text{ArcSinh}[a+b x]\right]}{\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} - \\
& \frac{1}{\sqrt{-1-a^2}} 2 b \left(-2 \text{ArcCos}\left[\pm a\right] \text{ArcTanh}\left[\frac{(-\pm a) \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] - \right. \\
& \left. (\pi - 2 \pm \text{ArcSinh}[a+b x]) \text{ArcTanh}\left[\frac{(\pm a) \tan\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] + \right. \\
& \left. \left(\text{ArcCos}\left[\pm a\right] + 2 \pm \text{ArcTanh}\left[\frac{(-\pm a) \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] + \right. \right. \\
& \left. \left. 2 \pm \text{ArcTanh}\left[\frac{(\pm a) \tan\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] \right) \text{Log}\left[\frac{\sqrt{-1-a^2} e^{-\frac{1}{2} \text{ArcSinh}[a+b x]}}{\sqrt{2} \sqrt{b x}}\right] + \right. \\
& \left. \left(\text{ArcCos}\left[\pm a\right] - 2 \pm \left(\text{ArcTanh}\left[\frac{(-\pm a) \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] + \text{ArcTanh}\left[\frac{(\pm a) \tan\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] \right) \right) \text{Log}\left[\frac{\pm \sqrt{-1-a^2} e^{\frac{1}{2} \text{ArcSinh}[a+b x]}}{\sqrt{2} \sqrt{b x}}\right] - \right. \\
& \left. \left(\text{ArcCos}\left[\pm a\right] + 2 \pm \text{ArcTanh}\left[\frac{(-\pm a) \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] \right) \right) \\
& \text{Log}\left[\left((\pm a) \left(a + \pm \left(-1 + \sqrt{-1-a^2}\right)\right) \left(\pm + \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right) / \right. \\
& \left. \left(\pm + a - \sqrt{-1-a^2} \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right] - \\
& \left. \left(\text{ArcCos}\left[\pm a\right] - 2 \pm \text{ArcTanh}\left[\frac{(-\pm a) \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] \right) \right) \\
& \text{Log}\left[\left((\pm a) \left(a - \pm \left(1 + \sqrt{-1-a^2}\right)\right) \left(-\pm + \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right) / \right. \\
& \left. \left(-\pm - a + \sqrt{-1-a^2} \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right] + \\
& \pm \left(\text{PolyLog}\left[2, -\left(\left((-\pm a + \sqrt{-1-a^2}) \left(\pm + a + \sqrt{-1-a^2} \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right) / \right. \right. \right. \\
& \left. \left. \left(-\pm - a + \sqrt{-1-a^2} \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right) \right] - \\
& \text{PolyLog}\left[2, \left(\left(\pm a + \sqrt{-1-a^2}\right) \left(\pm + a + \sqrt{-1-a^2} \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right) / \right. \\
& \left. \left(-\pm - a + \sqrt{-1-a^2} \cot\left[\frac{1}{4} (\pi + 2 \pm \text{ArcSinh}[a+b x])\right]\right) \right]
\end{aligned}$$

Problem 73: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\text{ArcSinh}[(a+b x)^2]}{x^3} dx$$

Optimal (type 4, 235 leaves, 14 steps):

$$\begin{aligned} & -\frac{b \sqrt{1+(a+b x)^2} \text{ArcSinh}[a+b x]}{(1+a^2) x} - \frac{\text{ArcSinh}[a+b x]^2}{2 x^2} + \\ & \frac{a b^2 \text{ArcSinh}[a+b x] \log \left[1-\frac{e^{\text{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} - \frac{a b^2 \text{ArcSinh}[a+b x] \log \left[1-\frac{e^{\text{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} + \\ & \frac{b^2 \log [x]}{1+a^2} + \frac{a b^2 \text{PolyLog}\left[2,\frac{e^{\text{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} - \frac{a b^2 \text{PolyLog}\left[2,\frac{e^{\text{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} \end{aligned}$$

Result (type 4, 925 leaves):

$$\begin{aligned} & -\frac{b \sqrt{1+(a+b x)^2} \text{ArcSinh}[a+b x]}{(1+a^2) x} - \frac{\text{ArcSinh}[a+b x]^2}{2 x^2} + \\ & \frac{i a b^2 \pi \text{ArcTanh}\left[\frac{-1-a \tanh \left[\frac{1}{2} \text{ArcSinh}[a+b x]\right]}{\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} + \frac{b^2 \log \left[-\frac{b x}{a}\right]}{1+a^2} - \\ & \frac{1}{(-1-a^2)^{3/2}} a b^2 \left(-2 \text{ArcCos}\left[\frac{1}{2} a\right] \text{ArcTanh}\left[\frac{(-\frac{1}{2}+a) \cot \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] - \right. \\ & \left. (\pi-2 i \text{ArcSinh}[a+b x]) \text{ArcTanh}\left[\frac{(\frac{1}{2}+a) \tan \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] + \right. \\ & \left. \left(\text{ArcCos}\left[\frac{1}{2} a\right]+2 i \text{ArcTanh}\left[\frac{(-\frac{1}{2}+a) \cot \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right]\right) \log \left[\frac{\sqrt{-1-a^2} e^{-\frac{1}{2} \text{ArcSinh}[a+b x]}}{\sqrt{2} \sqrt{b x}}\right] + \right. \\ & \left. 2 i \text{ArcTanh}\left[\frac{(\frac{1}{2}+a) \tan \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right] \right) \log \left[\frac{\frac{1}{2} \sqrt{-1-a^2} e^{\frac{1}{2} \text{ArcSinh}[a+b x]}}{\sqrt{2} \sqrt{b x}}\right] - \\ & \left(\text{ArcCos}\left[\frac{1}{2} a\right]-2 i \left(\text{ArcTanh}\left[\frac{(-\frac{1}{2}+a) \cot \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right]+\text{ArcTanh}\left[\frac{(\frac{1}{2}+a) \tan \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right]\right)\right) \log \left[\frac{\frac{1}{2} \sqrt{-1-a^2} e^{\frac{1}{2} \text{ArcSinh}[a+b x]}}{\sqrt{2} \sqrt{b x}}\right] - \\ & \left(\text{ArcCos}\left[\frac{1}{2} a\right]+2 i \text{ArcTanh}\left[\frac{(-\frac{1}{2}+a) \cot \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]}{\sqrt{-1-a^2}}\right]\right) \\ & \log \left[\left((\frac{1}{2}+a) \left(a+\frac{1}{2} \left(-1+\sqrt{-1-a^2}\right)\right) \left(\frac{1}{2}+\cot \left[\frac{1}{4} (\pi+2 i \text{ArcSinh}[a+b x])\right]\right)\right) \right] \end{aligned}$$

$$\begin{aligned}
& \left(\frac{i + a - \sqrt{-1 - a^2}}{4} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) - \\
& \left(\operatorname{ArcCos}[i a] - 2 i \operatorname{ArcTanh} \left[\frac{(-i + a) \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right]}{\sqrt{-1 - a^2}} \right] \right) \\
& \operatorname{Log} \left[\left((i + a) \left(a - i \left(1 + \sqrt{-1 - a^2} \right) \right) \left(-i + \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) \right) / \right. \\
& \left. \left(-i - a + \sqrt{-1 - a^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) \right] + \\
& i \left(\operatorname{PolyLog}[2, - \left(\left((-i a + \sqrt{-1 - a^2}) \left(i + a + \sqrt{-1 - a^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) \right) / \right. \right. \\
& \left. \left. \left(-i - a + \sqrt{-1 - a^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) \right] - \\
& \operatorname{PolyLog}[2, \left(\left(i a + \sqrt{-1 - a^2} \right) \left(i + a + \sqrt{-1 - a^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) \right) / \\
& \left. \left(-i - a + \sqrt{-1 - a^2} \operatorname{Cot} \left[\frac{1}{4} (\pi + 2 i \operatorname{ArcSinh}[a + b x]) \right] \right) \right)
\end{aligned}$$

Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSinh}[a + b x]^2}{x^4} dx$$

Optimal (type 4, 478 leaves, 40 steps):

$$\begin{aligned}
& -\frac{b^2}{3 (1 + a^2) x} - \frac{b \sqrt{1 + (a + b x)^2} \operatorname{ArcSinh}[a + b x]}{3 (1 + a^2) x^2} + \frac{a b^2 \sqrt{1 + (a + b x)^2} \operatorname{ArcSinh}[a + b x]}{(1 + a^2)^2 x} - \\
& \frac{\operatorname{ArcSinh}[a + b x]^2}{3 x^3} - \frac{a^2 b^3 \operatorname{ArcSinh}[a + b x] \operatorname{Log} \left[1 - \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a - \sqrt{1+a^2}} \right]}{(1 + a^2)^{5/2}} + \\
& \frac{b^3 \operatorname{ArcSinh}[a + b x] \operatorname{Log} \left[1 - \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a - \sqrt{1+a^2}} \right]}{3 (1 + a^2)^{3/2}} + \frac{a^2 b^3 \operatorname{ArcSinh}[a + b x] \operatorname{Log} \left[1 - \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a + \sqrt{1+a^2}} \right]}{(1 + a^2)^{5/2}} - \\
& \frac{b^3 \operatorname{ArcSinh}[a + b x] \operatorname{Log} \left[1 - \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a + \sqrt{1+a^2}} \right]}{3 (1 + a^2)^{3/2}} - \frac{a b^3 \operatorname{Log}[x]}{(1 + a^2)^2} - \frac{a^2 b^3 \operatorname{PolyLog}[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a - \sqrt{1+a^2}}]}{(1 + a^2)^{5/2}} + \\
& \frac{b^3 \operatorname{PolyLog}[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a - \sqrt{1+a^2}}]}{3 (1 + a^2)^{3/2}} + \frac{a^2 b^3 \operatorname{PolyLog}[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a + \sqrt{1+a^2}}]}{(1 + a^2)^{5/2}} - \frac{b^3 \operatorname{PolyLog}[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a + \sqrt{1+a^2}}]}{3 (1 + a^2)^{3/2}}
\end{aligned}$$

Result (type 4, 2153 leaves):

$$\begin{aligned}
& b^3 \left(-\frac{\sqrt{1 + (a + b x)^2} \operatorname{ArcSinh}[a + b x]}{3 (1 + a^2) b^2 x^2} - \right. \\
& \quad \left. \frac{\operatorname{ArcSinh}[a + b x]^2}{3 b^3 x^3} + \frac{-1 - a^2 + 3 a \sqrt{1 + (a + b x)^2} \operatorname{ArcSinh}[a + b x]}{3 (1 + a^2)^2 b x} - \right. \\
& \quad \left. \frac{a \operatorname{Log}\left[1 - \frac{a+b x}{a}\right]}{(1 + a^2)^2} - \frac{1}{3 (1 + a^2)^2} \left(-\frac{\frac{i \pi \operatorname{ArcTanh}\left[\frac{-1-a \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[a+b x]\right]}{\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} - \right. \right. \\
& \quad \left. \left. \frac{1}{\sqrt{-1-a^2}} \left(2 \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right) \operatorname{ArcTanh}\left[\frac{(-i-a) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right] - \right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcCos}\left[\frac{1}{2} a\right] \operatorname{ArcTanh}\left[\frac{(-i+a) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right] + \right. \right. \\
& \quad \left. \left. \left(\operatorname{ArcCos}\left[\frac{1}{2} a\right] - 2 \frac{i}{2} \left(\operatorname{ArcTanh}\left[\frac{(-i-a) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(-i+a) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right]\right) \right) \right) \\
& \quad \left(\operatorname{Log}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{-1-a^2} e^{-\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a+b x]\right)}}{\sqrt{b x}}\right] + \right. \\
& \quad \left. \left(\operatorname{ArcCos}\left[\frac{1}{2} a\right] + 2 \frac{i}{2} \left(\operatorname{ArcTanh}\left[\frac{(-i-a) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(-i+a) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right]\right) \right) \right) \\
& \quad \left(\operatorname{Log}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{-1-a^2} e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a+b x]\right)}}{\sqrt{b x}}\right] - \right. \\
& \quad \left. \left(\operatorname{ArcCos}\left[\frac{1}{2} a\right] + 2 \frac{i}{2} \operatorname{ArcTanh}\left[\frac{(-i+a) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1-a^2}}\right]\right) \right) \\
& \quad \left(\operatorname{Log}\left[1 - \left(i \left(-a - \frac{i}{2} \sqrt{-1-a^2}\right) \left(-i - a - \sqrt{-1-a^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a+b x]\right)\right]\right)\right] \right) / \\
& \quad \left(-i - a + \sqrt{-1-a^2} \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcSinh}[a+b x]\right)\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{ArcCos}[\text{i } a] + 2 \text{i } \text{ArcTanh}\left[\frac{(-\text{i } + a) \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] \right) \\
& \left(\text{Log}\left[1 - \left(\text{i } \left(-a + \text{i } \sqrt{-1 - a^2}\right) \left(-\text{i } - a - \sqrt{-1 - a^2} \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]\right)\right] \right) / \\
& \quad \left(-\text{i } - a + \sqrt{-1 - a^2} \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]\right] + \text{i } \left(\text{PolyLog}[2, \right. \\
& \quad \left. \left(\text{i } \left(-a - \text{i } \sqrt{-1 - a^2}\right) \left(-\text{i } - a - \sqrt{-1 - a^2} \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]\right)\right] / \\
& \quad \left(-\text{i } - a + \sqrt{-1 - a^2} \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]\right] - \text{PolyLog}[2, \\
& \quad \left. \left(\text{i } \left(-a + \text{i } \sqrt{-1 - a^2}\right) \left(-\text{i } - a - \sqrt{-1 - a^2} \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]\right)\right] \right) / \\
& \quad \left(-\text{i } - a + \sqrt{-1 - a^2} \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]\right] \right) \Bigg) + \\
& \frac{1}{3 (1 + a^2)^2} 2 a^2 \left(-\frac{\text{i } \pi \text{ArcTanh}\left[\frac{-1 - a \tanh\left[\frac{1}{2} \text{ArcSinh}[a + b x]\right]}{\sqrt{1 + a^2}} \right]}{\sqrt{1 + a^2}} - \frac{1}{\sqrt{-1 - a^2}} \right. \\
& \quad \left(2 \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right) \text{ArcTanh}\left[\frac{(-\text{i } - a) \cot\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] - \right. \\
& \quad \left. 2 \text{ArcCos}[\text{i } a] \text{ArcTanh}\left[\frac{(-\text{i } + a) \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] + \right. \\
& \quad \left. \left(\text{ArcCos}[\text{i } a] - 2 \text{i } \left(\text{ArcTanh}\left[\frac{(-\text{i } - a) \cot\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{ArcTanh}\left[\frac{(-\text{i } + a) \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] \right) \right) \right. \\
& \quad \left. \text{Log}\left[\frac{\left(\frac{1}{2} + \frac{\text{i }}{2}\right) \sqrt{-1 - a^2} e^{-\frac{1}{2} \text{i } \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)}}{\sqrt{b x}} \right] + \right. \\
& \quad \left. \left(\text{ArcCos}[\text{i } a] + 2 \text{i } \left(\text{ArcTanh}\left[\frac{(-\text{i } - a) \cot\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{ArcTanh}\left[\frac{(-\text{i } + a) \tan\left[\frac{1}{2} \left(\frac{\pi}{2} - \text{i } \text{ArcSinh}[a + b x]\right)\right]}{\sqrt{-1 - a^2}} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[\frac{\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{-1 - a^2} e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right)}}{\sqrt{b x}} \right] - \\
& \left(\text{ArcCos}[i a] + 2 i \text{ArcTanh} \left[\frac{(-i + a) \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right]}{\sqrt{-1 - a^2}} \right] \right) \\
& \text{Log} \left[1 - \left(i \left(-a - i \sqrt{-1 - a^2} \right) \left(-i - a - \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) \right) \right] / \\
& \left(-i - a + \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) + \\
& \left(-\text{ArcCos}[i a] + 2 i \text{ArcTanh} \left[\frac{(-i + a) \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right]}{\sqrt{-1 - a^2}} \right] \right) \\
& \text{Log} \left[1 - \left(i \left(-a + i \sqrt{-1 - a^2} \right) \left(-i - a - \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) \right) \right] / \\
& \left(-i - a + \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) + i \left(\text{PolyLog}[2, \right. \\
& \left. \left(i \left(-a - i \sqrt{-1 - a^2} \right) \left(-i - a - \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) \right) \right) / \\
& \left(-i - a + \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) - \text{PolyLog}[2, \\
& \left. \left(i \left(-a + i \sqrt{-1 - a^2} \right) \left(-i - a - \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) \right) \right) / \\
& \left(-i - a + \sqrt{-1 - a^2} \tan \left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcSinh}[a+b x] \right) \right] \right) \Bigg)
\end{aligned}$$

Problem 78: Unable to integrate problem.

$$\int \frac{\text{ArcSinh}[a+b x]^3}{x} dx$$

Optimal (type 4, 275 leaves, 13 steps):

$$\begin{aligned}
& -\frac{1}{4} \operatorname{ArcSinh}[a+b x]^4 + \operatorname{ArcSinh}[a+b x]^3 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + \\
& \operatorname{ArcSinh}[a+b x]^3 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] + 3 \operatorname{ArcSinh}[a+b x]^2 \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + \\
& 3 \operatorname{ArcSinh}[a+b x]^2 \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] - \\
& 6 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] - 6 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right] + \\
& 6 \operatorname{PolyLog}\left[4, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right] + 6 \operatorname{PolyLog}\left[4, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]
\end{aligned}$$

Result (type 8, 14 leaves):

$$\int \frac{\operatorname{ArcSinh}[a+b x]^3}{x} dx$$

Problem 79: Unable to integrate problem.

$$\int \frac{\operatorname{ArcSinh}[a+b x]^3}{x^2} dx$$

Optimal (type 4, 268 leaves, 13 steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcSinh}[a+b x]^3}{x} - \frac{3 b \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} + \\
& \frac{3 b \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} - \frac{6 b \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} + \\
& \frac{6 b \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} + \\
& \frac{6 b \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}} - \frac{6 b \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{\sqrt{1+a^2}}
\end{aligned}$$

Result (type 8, 14 leaves):

$$\int \frac{\operatorname{ArcSinh}[a+b x]^3}{x^2} dx$$

Problem 80: Unable to integrate problem.

$$\int \frac{\operatorname{ArcSinh}[a+b x]^3}{x^3} dx$$

Optimal (type 4, 514 leaves, 21 steps):

$$\begin{aligned}
& - \frac{3 b^2 \operatorname{ArcSinh}[a+b x]^2}{2 (1+a^2)} - \frac{3 b \sqrt{1+(a+b x)^2} \operatorname{ArcSinh}[a+b x]^2}{2 (1+a^2) x} - \frac{\operatorname{ArcSinh}[a+b x]^3}{2 x^2} + \\
& \frac{3 b^2 \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{1+a^2} + \frac{3 a b^2 \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{2 (1+a^2)^{3/2}} + \\
& \frac{3 b^2 \operatorname{ArcSinh}[a+b x] \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{1+a^2} - \frac{3 a b^2 \operatorname{ArcSinh}[a+b x]^2 \operatorname{Log}\left[1-\frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{2 (1+a^2)^{3/2}} + \\
& \frac{3 b^2 \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{1+a^2} + \frac{3 a b^2 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} + \\
& \frac{3 b^2 \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{1+a^2} - \frac{3 a b^2 \operatorname{ArcSinh}[a+b x] \operatorname{PolyLog}\left[2, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} - \\
& \frac{3 a b^2 \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a-\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}} + \frac{3 a b^2 \operatorname{PolyLog}\left[3, \frac{e^{\operatorname{ArcSinh}[a+b x]}}{a+\sqrt{1+a^2}}\right]}{(1+a^2)^{3/2}}
\end{aligned}$$

Result (type 8, 14 leaves):

$$\int \frac{\operatorname{ArcSinh}[a+b x]^3}{x^3} dx$$

Problem 94: Unable to integrate problem.

$$\int x^2 (a + b \operatorname{ArcSinh}[c + d x])^n dx$$

Optimal (type 4, 545 leaves, 22 steps):

$$\begin{aligned}
& \frac{1}{8 d^3} 3^{-1-n} e^{-\frac{3a}{b}} (a + b \operatorname{ArcSinh}[c + d x])^n \left(-\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \\
& \Gamma[1+n, -\frac{3(a + b \operatorname{ArcSinh}[c + d x])}{b}] - \frac{1}{d^3} 2^{-2-n} c e^{-\frac{2a}{b}} (a + b \operatorname{ArcSinh}[c + d x])^n \\
& \left(-\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \Gamma[1+n, -\frac{2(a + b \operatorname{ArcSinh}[c + d x])}{b}] - \frac{1}{8 d^3} \\
& e^{-\frac{a}{b}} (a + b \operatorname{ArcSinh}[c + d x])^n \left(-\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \Gamma[1+n, -\frac{a + b \operatorname{ArcSinh}[c + d x]}{b}] + \\
& \frac{1}{2 d^3} c^2 e^{-\frac{a}{b}} (a + b \operatorname{ArcSinh}[c + d x])^n \left(-\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \\
& \Gamma[1+n, -\frac{a + b \operatorname{ArcSinh}[c + d x]}{b}] + \frac{1}{8 d^3} \\
& e^{a/b} (a + b \operatorname{ArcSinh}[c + d x])^n \left(\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \Gamma[1+n, \frac{a + b \operatorname{ArcSinh}[c + d x]}{b}] - \frac{1}{2 d^3} \\
& c^2 e^{a/b} (a + b \operatorname{ArcSinh}[c + d x])^n \left(\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \Gamma[1+n, \frac{a + b \operatorname{ArcSinh}[c + d x]}{b}] - \\
& \frac{1}{d^3} 2^{-2-n} c e^{\frac{2a}{b}} (a + b \operatorname{ArcSinh}[c + d x])^n \left(\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \\
& \Gamma[1+n, \frac{2(a + b \operatorname{ArcSinh}[c + d x])}{b}] - \frac{1}{8 d^3} 3^{-1-n} e^{\frac{3a}{b}} (a + b \operatorname{ArcSinh}[c + d x])^n \\
& \left(\frac{a + b \operatorname{ArcSinh}[c + d x]}{b} \right)^{-n} \Gamma[1+n, \frac{3(a + b \operatorname{ArcSinh}[c + d x])}{b}]
\end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^2 (a + b \operatorname{ArcSinh}[c + d x])^n dx$$

Problem 126: Unable to integrate problem.

$$\int (c e + d e x)^m (a + b \operatorname{ArcSinh}[c + d x])^2 dx$$

Optimal (type 5, 187 leaves, 3 steps):

$$\begin{aligned}
& \frac{(e (c + d x))^{1+m} (a + b \operatorname{ArcSinh}[c + d x])^2}{d e (1 + m)} - \\
& \left(2 b (e (c + d x))^{2+m} (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -(c + d x)^2\right] \right) / \\
& (d e^2 (1 + m) (2 + m)) + \\
& \left(2 b^2 (e (c + d x))^{3+m} \operatorname{HypergeometricPFQ}\left[\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\}, \{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\}, -(c + d x)^2\right] \right) / \\
& (d e^3 (1 + m) (2 + m) (3 + m))
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int (c e + d e x)^m (a + b \operatorname{ArcSinh}[c + d x])^2 dx$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^2}{c e + d e x} dx$$

Optimal (type 4, 115 leaves, 8 steps):

$$-\frac{(a + b \operatorname{ArcSinh}[c + d x])^3}{3 b d e} + \frac{(a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} + \\ \frac{b (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} - \frac{b^2 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}]}{2 d e}$$

Result (type 4, 152 leaves):

$$\frac{1}{d e} \left(a^2 \operatorname{Log}[c + d x] + a b \right. \\ \left(\operatorname{ArcSinh}[c + d x] (\operatorname{ArcSinh}[c + d x] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c+d x]}]) - \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c+d x]}] \right) + \\ b^2 \left(\frac{i \pi^3}{24} - \frac{1}{3} \operatorname{ArcSinh}[c + d x]^3 + \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}] + \right. \\ \left. \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}] \right)$$

Problem 142: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^3}{c e + d e x} dx$$

Optimal (type 4, 155 leaves, 9 steps):

$$-\frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{4 b d e} + \frac{(a + b \operatorname{ArcSinh}[c + d x])^3 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} + \\ \frac{3 b (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}]}{2 d e} - \\ \frac{3 b^2 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}]}{2 d e} + \frac{3 b^3 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcSinh}[c+d x]}]}{4 d e}$$

Result (type 4, 256 leaves):

$$\frac{1}{64 d e} (64 a^3 \operatorname{Log}[c + d x] + 96 a^2 b (\operatorname{ArcSinh}[c + d x] (\operatorname{ArcSinh}[c + d x] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c+d x]}]) - \\ \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c+d x]}]) + \\ 8 a b^2 (\frac{i \pi^3}{8} - 8 \operatorname{ArcSinh}[c + d x]^3 + 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}] + \\ 24 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}] - 12 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}]) + \\ b^3 (\pi^4 - 16 \operatorname{ArcSinh}[c + d x]^4 + 64 \operatorname{ArcSinh}[c + d x]^3 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}] + \\ 96 \operatorname{ArcSinh}[c + d x]^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}] - \\ 96 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}] + 48 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcSinh}[c+d x]}]))$$

Problem 145: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^3}{(c e + d e x)^4} dx$$

Optimal (type 4, 261 leaves, 16 steps):

$$\begin{aligned} & -\frac{b^2 (a + b \operatorname{ArcSinh}[c + d x])}{d e^4 (c + d x)} - \frac{b \sqrt{1 + (c + d x)^2} (a + b \operatorname{ArcSinh}[c + d x])^2}{2 d e^4 (c + d x)^2} - \\ & \frac{(a + b \operatorname{ArcSinh}[c + d x])^3}{3 d e^4 (c + d x)^3} + \frac{b (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} - \\ & \frac{b^3 \operatorname{ArcTanh}[\sqrt{1 + (c + d x)^2}]}{d e^4} + \frac{b^2 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} - \\ & \frac{b^2 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} - \\ & \frac{b^3 \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} + \frac{b^3 \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} \end{aligned}$$

Result (type 4, 694 leaves):

$$\begin{aligned}
& - \frac{a^3}{3 d e^4 (c + d x)^3} - \frac{a^2 b \sqrt{1 + c^2 + 2 c d x + d^2 x^2}}{2 d e^4 (c + d x)^2} - \frac{a^2 b \operatorname{ArcSinh}[c + d x]}{d e^4 (c + d x)^3} - \frac{a^2 b \operatorname{Log}[c + d x]}{2 d e^4} + \\
& \frac{a^2 b \operatorname{Log}[1 + \sqrt{1 + c^2 + 2 c d x + d^2 x^2}]}{2 d e^4} + \frac{1}{8 d e^4} a b^2 \left(-8 \operatorname{PolyLog}[2, -e^{-\operatorname{ArcSinh}[c+d x]}] - \right. \\
& \left. \frac{1}{(c + d x)^3} 2 \left(-2 + 4 \operatorname{ArcSinh}[c + d x]^2 + 2 \operatorname{Cosh}[2 \operatorname{ArcSinh}[c + d x]] - 3 (c + d x) \operatorname{ArcSinh}[c + d x] \right. \right. \\
& \left. \left. \operatorname{Log}[1 - e^{-\operatorname{ArcSinh}[c+d x]}] + 3 (c + d x) \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 + e^{-\operatorname{ArcSinh}[c+d x]}] - \right. \right. \\
& \left. \left. 4 (c + d x)^3 \operatorname{PolyLog}[2, e^{-\operatorname{ArcSinh}[c+d x]}] + 2 \operatorname{ArcSinh}[c + d x] \operatorname{Sinh}[2 \operatorname{ArcSinh}[c + d x]] + \right. \right. \\
& \left. \left. \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 - e^{-\operatorname{ArcSinh}[c+d x]}] \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \right. \right. \\
& \left. \left. \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 + e^{-\operatorname{ArcSinh}[c+d x]}] \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] \right) \right) + \\
& \frac{1}{48 d e^4} b^3 \left(-24 \operatorname{ArcSinh}[c + d x] \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right] + 4 \operatorname{ArcSinh}[c + d x]^3 \right. \\
& \left. \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right] - 6 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]^2 - \right. \\
& \left. (c + d x) \operatorname{ArcSinh}[c + d x]^3 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]^4 - \right. \\
& \left. 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 - e^{-\operatorname{ArcSinh}[c+d x]}] + 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 + e^{-\operatorname{ArcSinh}[c+d x]}] + \right. \\
& \left. 48 \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]\right] - 48 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, -e^{-\operatorname{ArcSinh}[c+d x]}] + \right. \\
& \left. 48 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, e^{-\operatorname{ArcSinh}[c+d x]}] - 48 \operatorname{PolyLog}[3, -e^{-\operatorname{ArcSinh}[c+d x]}] + \right. \\
& \left. 48 \operatorname{PolyLog}[3, e^{-\operatorname{ArcSinh}[c+d x]}] - 6 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]^2 - \right. \\
& \left. 16 \operatorname{ArcSinh}[c + d x]^3 \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]^4 \right. \\
& \left. \left. + (c + d x)^3 \right) \right. \\
& \left. 24 \operatorname{ArcSinh}[c + d x] \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right] - 4 \operatorname{ArcSinh}[c + d x]^3 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right] \right)
\end{aligned}$$

Problem 151: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{c e + d e x} dx$$

Optimal (type 4, 186 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcSinh}[c + d x])^5}{5 b d e} + \frac{(a + b \operatorname{ArcSinh}[c + d x])^4 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} + \\
& \frac{2 b (a + b \operatorname{ArcSinh}[c + d x])^3 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} - \\
& \frac{3 b^2 (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} + \\
& \frac{3 b^3 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[4, e^{2 \operatorname{ArcSinh}[c+d x]}]}{d e} - \frac{3 b^4 \operatorname{PolyLog}[5, e^{2 \operatorname{ArcSinh}[c+d x]}]}{2 d e}
\end{aligned}$$

Result (type 4, 390 leaves):

$$\begin{aligned}
& \frac{1}{16 d e} \left(16 a^4 \operatorname{Log}[c + d x] + 32 a^3 b (\operatorname{ArcSinh}[c + d x] (\operatorname{ArcSinh}[c + d x] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c+d x]}]) - \right. \\
& \quad \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c+d x]}]) + \\
& \quad 4 a^2 b^2 (\frac{1}{2} \pi^3 - 8 \operatorname{ArcSinh}[c + d x]^3 + 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}] + \\
& \quad 24 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}] - 12 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}]) + \\
& \quad a b^3 (\pi^4 - 16 \operatorname{ArcSinh}[c + d x]^4 + 64 \operatorname{ArcSinh}[c + d x]^3 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}] + \\
& \quad 96 \operatorname{ArcSinh}[c + d x]^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}] - \\
& \quad 96 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}] + 48 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcSinh}[c+d x]}]) + \\
& \quad 16 b^4 \left(-\frac{\frac{1}{160} \pi^5}{160} - \frac{1}{5} \operatorname{ArcSinh}[c + d x]^5 + \operatorname{ArcSinh}[c + d x]^4 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c+d x]}] + \right. \\
& \quad 2 \operatorname{ArcSinh}[c + d x]^3 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c+d x]}] - \\
& \quad 3 \operatorname{ArcSinh}[c + d x]^2 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c+d x]}] + \\
& \quad \left. 3 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[4, e^{2 \operatorname{ArcSinh}[c+d x]}] - \frac{3}{2} \operatorname{PolyLog}[5, e^{2 \operatorname{ArcSinh}[c+d x]}] \right)
\end{aligned}$$

Problem 152: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{(c e + d e x)^2} d x$$

Optimal (type 4, 234 leaves, 13 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{d e^2 (c + d x)} - \frac{8 b (a + b \operatorname{ArcSinh}[c + d x])^3 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2} - \\
& \frac{12 b^2 (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2} + \\
& \frac{12 b^2 (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2} + \\
& \frac{24 b^3 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2} - \\
& \frac{24 b^3 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2} - \\
& \frac{24 b^4 \operatorname{PolyLog}[4, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2} + \frac{24 b^4 \operatorname{PolyLog}[4, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^2}
\end{aligned}$$

Result (type 4, 510 leaves) :

$$\begin{aligned}
& \frac{1}{2 d e^2} \left(-\frac{2 a^4}{c + d x} - 8 a^3 b \left(\frac{\text{ArcSinh}[c + d x]}{c + d x} + \right. \right. \\
& \quad \left. \left. \text{Log}\left[\frac{1}{2} (c + d x) \text{Csch}\left[\frac{1}{2} \text{ArcSinh}[c + d x]\right]\right] - \text{Log}\left[\text{Sinh}\left[\frac{1}{2} \text{ArcSinh}[c + d x]\right]\right] \right) + 12 a^2 b^2 \right. \\
& \quad \left(\text{ArcSinh}[c + d x] \left(-\frac{\text{ArcSinh}[c + d x]}{c + d x} + 2 \text{Log}\left[1 - e^{-\text{ArcSinh}[c+d x]}\right] - 2 \text{Log}\left[1 + e^{-\text{ArcSinh}[c+d x]}\right] \right) + \right. \\
& \quad \left. \left. 2 \text{PolyLog}\left[2, -e^{-\text{ArcSinh}[c+d x]}\right] - 2 \text{PolyLog}\left[2, e^{-\text{ArcSinh}[c+d x]}\right] \right) + \right. \\
& \quad \left. 8 a b^3 \left(-\frac{\text{ArcSinh}[c + d x]^3}{c + d x} + 3 \text{ArcSinh}[c + d x]^2 \text{Log}\left[1 - e^{-\text{ArcSinh}[c+d x]}\right] - \right. \right. \\
& \quad \left. \left. 3 \text{ArcSinh}[c + d x]^2 \text{Log}\left[1 + e^{-\text{ArcSinh}[c+d x]}\right] + 6 \text{ArcSinh}[c + d x] \text{PolyLog}\left[2, -e^{-\text{ArcSinh}[c+d x]}\right] - \right. \right. \\
& \quad \left. \left. 6 \text{ArcSinh}[c + d x] \text{PolyLog}\left[2, e^{-\text{ArcSinh}[c+d x]}\right] + \right. \right. \\
& \quad \left. \left. 6 \text{PolyLog}\left[3, -e^{-\text{ArcSinh}[c+d x]}\right] - 6 \text{PolyLog}\left[3, e^{-\text{ArcSinh}[c+d x]}\right] \right) + \right. \\
& \quad \left. b^4 \left(\pi^4 - 2 \text{ArcSinh}[c + d x]^4 - \frac{2 \text{ArcSinh}[c + d x]^4}{c + d x} - 8 \text{ArcSinh}[c + d x]^3 \text{Log}\left[1 + e^{-\text{ArcSinh}[c+d x]}\right] + \right. \right. \\
& \quad \left. \left. 8 \text{ArcSinh}[c + d x]^3 \text{Log}\left[1 - e^{\text{ArcSinh}[c+d x]}\right] + 24 \text{ArcSinh}[c + d x]^2 \text{PolyLog}\left[2, -e^{-\text{ArcSinh}[c+d x]}\right] + \right. \right. \\
& \quad \left. \left. 24 \text{ArcSinh}[c + d x]^2 \text{PolyLog}\left[2, e^{\text{ArcSinh}[c+d x]}\right] + 48 \text{ArcSinh}[c + d x] \text{PolyLog}\left[3, -e^{-\text{ArcSinh}[c+d x]}\right] - 48 \text{ArcSinh}[c + d x] \text{PolyLog}\left[3, e^{\text{ArcSinh}[c+d x]}\right] + \right. \right. \\
& \quad \left. \left. 48 \text{PolyLog}\left[4, -e^{-\text{ArcSinh}[c+d x]}\right] + 48 \text{PolyLog}\left[4, e^{\text{ArcSinh}[c+d x]}\right] \right) \right)
\end{aligned}$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \text{ArcSinh}[c + d x])^4}{(c e + d e x)^3} dx$$

Optimal (type 4, 186 leaves, 10 steps) :

$$\begin{aligned}
& -\frac{2 b (a + b \text{ArcSinh}[c + d x])^3}{d e^3} - \frac{2 b \sqrt{1 + (c + d x)^2} (a + b \text{ArcSinh}[c + d x])^3}{d e^3 (c + d x)} - \\
& \frac{(a + b \text{ArcSinh}[c + d x])^4}{2 d e^3 (c + d x)^2} + \frac{6 b^2 (a + b \text{ArcSinh}[c + d x])^2 \text{Log}\left[1 - e^{2 \text{ArcSinh}[c+d x]}\right]}{d e^3} + \\
& \frac{6 b^3 (a + b \text{ArcSinh}[c + d x]) \text{PolyLog}\left[2, e^{2 \text{ArcSinh}[c+d x]}\right]}{d e^3} - \frac{3 b^4 \text{PolyLog}\left[3, e^{2 \text{ArcSinh}[c+d x]}\right]}{d e^3}
\end{aligned}$$

Result (type 4, 360 leaves) :

$$\begin{aligned}
& \frac{1}{4 d e^3} \left(-\frac{2 a^4}{(c + d x)^2} - \frac{8 a^3 b \sqrt{1 + (c + d x)^2}}{c + d x} - \frac{8 a^3 b \operatorname{ArcSinh}[c + d x]}{(c + d x)^2} - \frac{2 b^4 \operatorname{ArcSinh}[c + d x]^4}{(c + d x)^2} + \right. \\
& 24 a^2 b^2 \left(-\frac{\sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x]}{c + d x} - \frac{\operatorname{ArcSinh}[c + d x]^2}{2 (c + d x)^2} + \operatorname{Log}[c + d x] \right) + \\
& 8 a b^3 \left(\operatorname{ArcSinh}[c + d x] \left(3 \operatorname{ArcSinh}[c + d x] - \frac{3 \sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x]}{c + d x} - \right. \right. \\
& \left. \left. \frac{\operatorname{ArcSinh}[c + d x]^2}{(c + d x)^2} + 6 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcSinh}[c+d x]}\right] \right) - 3 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSinh}[c+d x]}\right] \right) + \\
& b^4 \left(\frac{8 \sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x]^3}{c + d x} + \right. \\
& 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}[c+d x]}\right] + \\
& \left. \left. 24 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}[c+d x]}\right] - 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcSinh}[c+d x]}\right] \right) \right)
\end{aligned}$$

Problem 154: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{(c e + d e x)^4} dx$$

Optimal (type 4, 385 leaves, 21 steps):

$$\begin{aligned}
& - \frac{2 b^2 (a + b \operatorname{ArcSinh}[c + d x])^2}{d e^4 (c + d x)} - \frac{2 b \sqrt{1 + (c + d x)^2} (a + b \operatorname{ArcSinh}[c + d x])^3}{3 d e^4 (c + d x)^2} - \\
& \frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{3 d e^4 (c + d x)^3} - \frac{8 b^3 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} + \\
& \frac{4 b (a + b \operatorname{ArcSinh}[c + d x])^3 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c+d x]}]}{3 d e^4} - \frac{4 b^4 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} + \\
& \frac{2 b^2 (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} + \\
& \frac{4 b^4 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} - \frac{2 b^2 (a + b \operatorname{ArcSinh}[c + d x])^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} - \\
& \frac{4 b^3 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} + \\
& \frac{4 b^3 (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} + \\
& \frac{4 b^4 \operatorname{PolyLog}[4, -e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4} - \frac{4 b^4 \operatorname{PolyLog}[4, e^{\operatorname{ArcSinh}[c+d x]}]}{d e^4}
\end{aligned}$$

Result (type 4, 1198 leaves):

$$\begin{aligned}
& - \frac{a^4}{3 d e^4 (c + d x)^3} + \\
& \frac{1}{4 d e^4} a^2 b^2 \left(-8 \operatorname{PolyLog}[2, -e^{-\operatorname{ArcSinh}[c+d x]}] - \frac{1}{(c + d x)^3} 2 \left(-2 + 4 \operatorname{ArcSinh}[c + d x]^2 + \right. \right. \\
& 2 \operatorname{Cosh}[2 \operatorname{ArcSinh}[c + d x]] - 3 (c + d x) \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 - e^{-\operatorname{ArcSinh}[c+d x]}] + \\
& 3 (c + d x) \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 + e^{-\operatorname{ArcSinh}[c+d x]}] - 4 (c + d x)^3 \\
& \operatorname{PolyLog}[2, e^{-\operatorname{ArcSinh}[c+d x]}] + 2 \operatorname{ArcSinh}[c + d x] \operatorname{Sinh}[2 \operatorname{ArcSinh}[c + d x]] + \\
& \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 - e^{-\operatorname{ArcSinh}[c+d x]}] \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \\
& \left. \left. \operatorname{ArcSinh}[c + d x] \operatorname{Log}[1 + e^{-\operatorname{ArcSinh}[c+d x]}] \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] \right) \right) + \\
& \frac{1}{12 d e^4} a b^3 \left(-24 \operatorname{ArcSinh}[c + d x] \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right] + \right. \\
& 4 \operatorname{ArcSinh}[c + d x]^3 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right] - \\
& 6 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]^2 - \\
& (c + d x) \operatorname{ArcSinh}[c + d x]^3 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]^4 - \\
& 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 - e^{-\operatorname{ArcSinh}[c+d x]}] + \\
& 24 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Log}[1 + e^{-\operatorname{ArcSinh}[c+d x]}] + 48 \operatorname{Log}[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c + d x]\right]] - \\
& 48 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, -e^{-\operatorname{ArcSinh}[c+d x]}] + \\
& 48 \operatorname{ArcSinh}[c + d x] \operatorname{PolyLog}[2, e^{-\operatorname{ArcSinh}[c+d x]}] - 48 \operatorname{PolyLog}[3, -e^{-\operatorname{ArcSinh}[c+d x]}] +
\end{aligned}$$

$$\begin{aligned}
& \frac{48 \operatorname{PolyLog}[3, e^{-\operatorname{ArcSinh}[c+d x]}] - 6 \operatorname{ArcSinh}[c+d x]^2 \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 - \\
& \frac{16 \operatorname{ArcSinh}[c+d x]^3 \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^4}{(c+d x)^3} + \\
& 24 \operatorname{ArcSinh}[c+d x] \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] - 4 \operatorname{ArcSinh}[c+d x]^3 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] \Big) + \\
& \frac{1}{24 d e^4} b^4 \left(-2 \pi^4 + 4 \operatorname{ArcSinh}[c+d x]^4 - 24 \operatorname{ArcSinh}[c+d x]^2 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] + \right. \\
& 2 \operatorname{ArcSinh}[c+d x]^4 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] - 4 \operatorname{ArcSinh}[c+d x]^3 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 - \\
& \frac{1}{2} (c+d x) \operatorname{ArcSinh}[c+d x]^4 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^4 + \\
& 96 \operatorname{ArcSinh}[c+d x] \operatorname{Log}\left[1 - e^{-\operatorname{ArcSinh}[c+d x]}\right] - 96 \operatorname{ArcSinh}[c+d x] \operatorname{Log}\left[1 + e^{-\operatorname{ArcSinh}[c+d x]}\right] + \\
& 16 \operatorname{ArcSinh}[c+d x]^3 \operatorname{Log}\left[1 + e^{-\operatorname{ArcSinh}[c+d x]}\right] - 16 \operatorname{ArcSinh}[c+d x]^3 \operatorname{Log}\left[1 - e^{\operatorname{ArcSinh}[c+d x]}\right] - \\
& 48 (-2 + \operatorname{ArcSinh}[c+d x]^2) \operatorname{PolyLog}[2, -e^{-\operatorname{ArcSinh}[c+d x]}] - \\
& 96 \operatorname{PolyLog}[2, e^{-\operatorname{ArcSinh}[c+d x]}] - 48 \operatorname{ArcSinh}[c+d x]^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c+d x]}] - \\
& 96 \operatorname{ArcSinh}[c+d x] \operatorname{PolyLog}[3, -e^{-\operatorname{ArcSinh}[c+d x]}] + \\
& 96 \operatorname{ArcSinh}[c+d x] \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c+d x]}] - 96 \operatorname{PolyLog}[4, -e^{-\operatorname{ArcSinh}[c+d x]}] - \\
& 96 \operatorname{PolyLog}[4, e^{\operatorname{ArcSinh}[c+d x]}] - 4 \operatorname{ArcSinh}[c+d x]^3 \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 - \\
& \frac{8 \operatorname{ArcSinh}[c+d x]^4 \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^4}{(c+d x)^3} + 24 \operatorname{ArcSinh}[c+d x]^2 \\
& \left. \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] - 2 \operatorname{ArcSinh}[c+d x]^4 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] \right) + \\
& \frac{1}{d e^4} 4 a^3 b \left(\frac{1}{12} \operatorname{ArcSinh}[c+d x] \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] - \frac{1}{24} \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 - \right. \\
& \frac{1}{24} \operatorname{ArcSinh}[c+d x] \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 + \\
& \frac{1}{6} \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]\right] - \frac{1}{6} \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]\right] - \\
& \frac{1}{24} \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 - \frac{1}{12} \operatorname{ArcSinh}[c+d x] \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] - \\
& \left. \frac{1}{24} \operatorname{ArcSinh}[c+d x] \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right]^2 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSinh}[c+d x]\right] \right)
\end{aligned}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int (c e + d e x)^2 (a + b \operatorname{ArcSinh}[c+d x])^{7/2} dx$$

Optimal (type 4, 481 leaves, 35 steps):

$$\begin{aligned}
& \frac{175 b^3 e^2 \sqrt{1 + (c + d x)^2} \sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{54 d} - \\
& \frac{35 b^3 e^2 (c + d x)^2 \sqrt{1 + (c + d x)^2} \sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{216 d} - \\
& \frac{35 b^2 e^2 (c + d x) (a + b \operatorname{ArcSinh}[c + d x])^{3/2}}{18 d} + \frac{35 b^2 e^2 (c + d x)^3 (a + b \operatorname{ArcSinh}[c + d x])^{3/2}}{108 d} + \\
& \frac{7 b e^2 \sqrt{1 + (c + d x)^2} (a + b \operatorname{ArcSinh}[c + d x])^{5/2}}{9 d} - \\
& \frac{7 b e^2 (c + d x)^2 \sqrt{1 + (c + d x)^2} (a + b \operatorname{ArcSinh}[c + d x])^{5/2}}{18 d} + \\
& \frac{e^2 (c + d x)^3 (a + b \operatorname{ArcSinh}[c + d x])^{7/2}}{3 d} - \frac{105 b^{7/2} e^2 e^{a/b} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{a+b} \operatorname{ArcSinh}[c+d x]}{\sqrt{b}}\right]}{128 d} + \\
& \frac{35 b^{7/2} e^2 e^{\frac{3 a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{a+b} \operatorname{ArcSinh}[c+d x]}{\sqrt{b}}\right]}{3456 d} - \frac{105 b^{7/2} e^2 e^{-\frac{a}{b}} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{a+b} \operatorname{ArcSinh}[c+d x]}{\sqrt{b}}\right]}{128 d} + \\
& \frac{35 b^{7/2} e^2 e^{-\frac{3 a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a+b} \operatorname{ArcSinh}[c+d x]}{\sqrt{b}}\right]}{3456 d}
\end{aligned}$$

Result (type 4, 1095 leaves) :

$$\begin{aligned}
& - \frac{1}{10368 d} e^2 \left(2592 a^3 c \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + 22680 a b^2 c \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \right. \\
& 2592 a^3 d x \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + 22680 a b^2 d x \sqrt{a + b \operatorname{ArcSinh}[c + d x]} - \\
& 9072 a^2 b \sqrt{1 + c^2 + 2 c d x + d^2 x^2} \sqrt{a + b \operatorname{ArcSinh}[c + d x]} - 34020 b^3 \sqrt{1 + c^2 + 2 c d x + d^2 x^2} \\
& \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + 7776 a^2 b c \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \\
& 22680 b^3 c \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + 7776 a^2 b d x \operatorname{ArcSinh}[c + d x] \\
& \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + 22680 b^3 d x \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} - \\
& 18144 a b^2 \sqrt{1 + c^2 + 2 c d x + d^2 x^2} \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \\
& 7776 a b^2 c \operatorname{ArcSinh}[c + d x]^2 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \\
& 7776 a b^2 d x \operatorname{ArcSinh}[c + d x]^2 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} - \\
& 9072 b^3 \sqrt{1 + c^2 + 2 c d x + d^2 x^2} \operatorname{ArcSinh}[c + d x]^2 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \\
& 2592 b^3 c \operatorname{ArcSinh}[c + d x]^3 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \\
& 2592 b^3 d x \operatorname{ArcSinh}[c + d x]^3 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} + \\
& 1008 a^2 b \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcSinh}[c + d x]] + \\
& 420 b^3 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcSinh}[c + d x]] + \\
& 2016 a b^2 \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcSinh}[c + d x]] + \\
& 1008 b^3 \operatorname{ArcSinh}[c + d x]^2 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Cosh}[3 \operatorname{ArcSinh}[c + d x]] + \\
& 8505 b^{7/2} \sqrt{\pi} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{Erfi}\left[\frac{\sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{\sqrt{b}}\right] - \\
& 35 b^{7/2} \sqrt{3 \pi} \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{\sqrt{b}}\right] - \\
& 8505 b^{7/2} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{\sqrt{b}}\right] \operatorname{Sinh}\left[\frac{a}{b}\right] + \\
& 8505 b^{7/2} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{\sqrt{b}}\right] \left(\operatorname{Cosh}\left[\frac{a}{b}\right] + \operatorname{Sinh}\left[\frac{a}{b}\right]\right) + \\
& 35 b^{7/2} \sqrt{3 \pi} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{\sqrt{b}}\right] \operatorname{Sinh}\left[\frac{3 a}{b}\right] - \\
& 35 b^{7/2} \sqrt{3 \pi} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{a + b \operatorname{ArcSinh}[c + d x]}}{\sqrt{b}}\right] \left(\operatorname{Cosh}\left[\frac{3 a}{b}\right] + \operatorname{Sinh}\left[\frac{3 a}{b}\right]\right) - \\
& 864 a^3 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \\
& 840 a b^2 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \\
& 2592 a^2 b \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \\
& 840 b^3 \operatorname{ArcSinh}[c + d x] \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \\
& 2592 a b^2 \operatorname{ArcSinh}[c + d x]^2 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] - \\
& \left. 864 b^3 \operatorname{ArcSinh}[c + d x]^3 \sqrt{a + b \operatorname{ArcSinh}[c + d x]} \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] \right)
\end{aligned}$$

Problem 228: Result unnecessarily involves imaginary or complex numbers.

$$\int (c e + d e x)^{7/2} (a + b \operatorname{ArcSinh}[c + d x]) dx$$

Optimal (type 4, 298 leaves, 8 steps) :

$$\begin{aligned} & \frac{28 b e^2 (e (c + d x))^{3/2} \sqrt{1 + (c + d x)^2}}{405 d} - \frac{4 b (e (c + d x))^{7/2} \sqrt{1 + (c + d x)^2}}{81 d} - \\ & \frac{28 b e^3 \sqrt{e (c + d x)} \sqrt{1 + (c + d x)^2}}{135 d (1 + c + d x)} + \frac{2 (e (c + d x))^{9/2} (a + b \text{ArcSinh}[c + d x])}{9 d e} + \\ & \frac{28 b e^{7/2} (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{135 d \sqrt{1 + (c + d x)^2}} - \\ & \frac{14 b e^{7/2} (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{135 d \sqrt{1 + (c + d x)^2}} \end{aligned}$$

Result (type 4, 150 leaves) :

$$\begin{aligned} & \frac{1}{135 d} (e (c + d x))^{7/2} \\ & \left(30 a (c + d x) - \frac{4 b (-7 + 5 c^2 + 10 c d x + 5 d^2 x^2) \sqrt{1 + (c + d x)^2}}{3 (c + d x)^2} + 30 b (c + d x) \text{ArcSinh}[c + d x] + \right. \\ & \left. \frac{1}{(c + d x)^{7/2}} 28 (-1)^{3/4} b \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] - \right. \right. \\ & \left. \left. \text{EllipticF}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] \right) \right) \end{aligned}$$

Problem 229: Result unnecessarily involves imaginary or complex numbers.

$$\int (c e + d e x)^{5/2} (a + b \text{ArcSinh}[c + d x]) dx$$

Optimal (type 4, 177 leaves, 6 steps) :

$$\begin{aligned} & \frac{20 b e^2 \sqrt{e (c + d x)} \sqrt{1 + (c + d x)^2}}{147 d} - \\ & \frac{4 b (e (c + d x))^{5/2} \sqrt{1 + (c + d x)^2}}{49 d} + \frac{2 (e (c + d x))^{7/2} (a + b \text{ArcSinh}[c + d x])}{7 d e} - \\ & \frac{10 b e^{5/2} (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{147 d \sqrt{1 + (c + d x)^2}} \end{aligned}$$

Result (type 4, 149 leaves) :

$$\frac{1}{147 d} (e (c + d x))^{5/2} \left(42 a (c + d x) - \frac{4 b (-5 + 3 c^2 + 6 c d x + 3 d^2 x^2) \sqrt{1 + (c + d x)^2}}{(c + d x)^2} + 42 b (c + d x) \text{ArcSinh}[c + d x] - \right. \\ \left. \frac{20 (-1)^{1/4} b \sqrt{1 + (c + d x)^2} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c+d x}}\right], -1\right]}{(c + d x)^{7/2} \sqrt{1 + \frac{1}{(c+d x)^2}}} \right)$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int (c e + d e x)^{3/2} (a + b \text{ArcSinh}[c + d x]) dx$$

Optimal (type 4, 261 leaves, 7 steps) :

$$-\frac{4 b (e (c + d x))^{3/2} \sqrt{1 + (c + d x)^2}}{25 d} + \\ \frac{12 b e \sqrt{e (c + d x)} \sqrt{1 + (c + d x)^2}}{25 d (1 + c + d x)} + \frac{2 (e (c + d x))^{5/2} (a + b \text{ArcSinh}[c + d x])}{5 d e} - \\ \frac{12 b e^{3/2} (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c+d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{25 d \sqrt{1 + (c + d x)^2}} + \\ \frac{6 b e^{3/2} (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c+d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{25 d \sqrt{1 + (c + d x)^2}}$$

Result (type 4, 145 leaves) :

$$\frac{1}{25 d (c + d x)^{3/2}} 2 (e (c + d x))^{3/2} \left((c + d x)^{3/2} \left(5 a (c + d x) - 2 b \sqrt{1 + c^2 + 2 c d x + d^2 x^2} + 5 b (c + d x) \text{ArcSinh}[c + d x] \right) - \right. \\ \left. 6 (-1)^{3/4} b \text{EllipticE}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] + \right. \\ \left. 6 (-1)^{3/4} b \text{EllipticF}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] \right)$$

Problem 231: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcSinh}[c + d x]) dx$$

Optimal (type 4, 142 leaves, 5 steps) :

$$\begin{aligned} & -\frac{4 b \sqrt{e (c + d x)} \sqrt{1 + (c + d x)^2}}{9 d} + \frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x])}{3 d e} + \\ & \frac{2 b \sqrt{e} (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{9 d \sqrt{1 + (c + d x)^2}} \end{aligned}$$

Result (type 4, 122 leaves) :

$$\begin{aligned} & \frac{1}{9 d} 2 \sqrt{e (c + d x)} \left(3 a (c + d x) - 2 b \sqrt{1 + (c + d x)^2} + 3 b (c + d x) \operatorname{ArcSinh}[c + d x] + \right. \\ & \left. \frac{2 (-1)^{1/4} b \sqrt{1 + (c + d x)^2} \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c + d x}}\right], -1\right]}{(c + d x)^{3/2} \sqrt{1 + \frac{1}{(c + d x)^2}}} \right) \end{aligned}$$

Problem 232: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcSinh}[c + d x]}{\sqrt{c e + d e x}} dx$$

Optimal (type 4, 223 leaves, 6 steps) :

$$\begin{aligned} & -\frac{4 b \sqrt{e (c + d x)} \sqrt{1 + (c + d x)^2}}{d e (1 + c + d x)} + \frac{2 \sqrt{e (c + d x)} (a + b \operatorname{ArcSinh}[c + d x])}{d e} + \\ & \frac{4 b (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{d \sqrt{e} \sqrt{1 + (c + d x)^2}} - \\ & \frac{2 b (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{d \sqrt{e} \sqrt{1 + (c + d x)^2}} \end{aligned}$$

Result (type 4, 111 leaves) :

$$\frac{1}{d \sqrt{e (c + d x)}} \left(2 (c + d x) (a + b \text{ArcSinh}[c + d x]) + \right. \\ \left. 4 (-1)^{3/4} b \sqrt{c + d x} \text{EllipticE}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] - \right. \\ \left. 4 (-1)^{3/4} b \sqrt{c + d x} \text{EllipticF}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right]\right)$$

Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \text{ArcSinh}[c + d x]}{(c e + d e x)^{3/2}} dx$$

Optimal (type 4, 106 leaves, 4 steps):

$$-\frac{2 (a + b \text{ArcSinh}[c + d x])}{d e \sqrt{e (c + d x)}} + \frac{2 b (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{d e^{3/2} \sqrt{1 + (c + d x)^2}}$$

Result (type 4, 104 leaves):

$$\frac{1}{d (e (c + d x))^{3/2}} 2 \left(-a (c + d x) - b (c + d x) \text{ArcSinh}[c + d x] + \frac{1}{\sqrt{1 + \frac{1}{(c + d x)^2}}} \right. \\ \left. 2 (-1)^{1/4} b \sqrt{c + d x} \sqrt{1 + (c + d x)^2} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c + d x}}\right], -1\right] \right)$$

Problem 234: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \text{ArcSinh}[c + d x]}{(c e + d e x)^{5/2}} dx$$

Optimal (type 4, 266 leaves, 7 steps):

$$\begin{aligned}
& - \frac{4 b \sqrt{1 + (c + d x)^2}}{3 d e^2 \sqrt{e (c + d x)}} + \frac{4 b \sqrt{e (c + d x)} \sqrt{1 + (c + d x)^2}}{3 d e^3 (1 + c + d x)} - \frac{2 (a + b \text{ArcSinh}[c + d x])}{3 d e (e (c + d x))^{3/2}} - \\
& \frac{4 b (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{3 d e^{5/2} \sqrt{1 + (c + d x)^2}} + \\
& \frac{2 b (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{3 d e^{5/2} \sqrt{1 + (c + d x)^2}}
\end{aligned}$$

Result (type 4, 160 leaves) :

$$\begin{aligned}
& - \left(\left(2 \left(a + 2 b c \sqrt{1 + c^2 + 2 c d x + d^2 x^2} + 2 b d x \sqrt{1 + c^2 + 2 c d x + d^2 x^2} + b \text{ArcSinh}[c + d x] + \right. \right. \right. \\
& \left. \left. \left. 2 (-1)^{3/4} b (c + d x)^{3/2} \text{EllipticE}\left[\text{i ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] - 2 (-1)^{3/4} b \right. \right. \\
& \left. \left. (c + d x)^{3/2} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] \right) \right) / \left(3 d e (e (c + d x))^{3/2} \right)
\end{aligned}$$

Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \text{ArcSinh}[c + d x]}{(c e + d e x)^{7/2}} dx$$

Optimal (type 4, 145 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{4 b \sqrt{1 + (c + d x)^2}}{15 d e^2 (e (c + d x))^{3/2}} - \frac{2 (a + b \text{ArcSinh}[c + d x])}{5 d e (e (c + d x))^{5/2}} - \\
& \frac{2 b (1 + c + d x) \sqrt{\frac{1 + (c + d x)^2}{(1 + c + d x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{e (c + d x)}}{\sqrt{e}}\right], \frac{1}{2}\right]}{15 d e^{7/2} \sqrt{1 + (c + d x)^2}}
\end{aligned}$$

Result (type 4, 167 leaves) :

$$\begin{aligned}
& - \left(\left(2 \left(\sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}} \left(3 a + 2 b (c + d x) \sqrt{1 + c^2 + 2 c d x + d^2 x^2} + 3 b \text{ArcSinh}[c + d x] \right) + \right. \right. \right. \\
& \left. \left. \left. 2 (-1)^{1/4} b (c + d x)^{3/2} \sqrt{1 + c^2 + 2 c d x + d^2 x^2} \text{EllipticF}\left[\text{i ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c + d x}}\right], -1\right] \right) \right) / \\
& \left(15 d e (e (c + d x))^{5/2} \sqrt{1 + \frac{1}{(c + d x)^2}} \right)
\end{aligned}$$

Problem 236: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int (c e + d e x)^{7/2} (a + b \operatorname{ArcSinh}[c + d x])^2 dx$$

Optimal (type 5, 134 leaves, 3 steps) :

$$\frac{2 (e (c + d x))^{9/2} (a + b \operatorname{ArcSinh}[c + d x])^2}{9 d e} - \frac{1}{99 d e^2}$$

$$8 b (e (c + d x))^{11/2} (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{11}{4}, \frac{15}{4}, -(c + d x)^2\right] +$$

$$\frac{1}{1287 d e^3} 16 b^2 (e (c + d x))^{13/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{13}{4}, \frac{13}{4}\right\}, \left\{\frac{15}{4}, \frac{17}{4}\right\}, -(c + d x)^2\right]$$

Result (type 5, 269 leaves) :

$$\frac{1}{9 d} (e (c + d x))^{7/2}$$

$$\left(2 a^2 (c + d x) + 4 a b (c + d x) \operatorname{ArcSinh}[c + d x] - \frac{1}{45 (c + d x)^{7/2}} 8 a b \left((c + d x)^{3/2} \sqrt{1 + (c + d x)^2}\right.\right.$$

$$\left.\left.(-7 + 5 (c + d x)^2) + 21 (-1)^{3/4} \left(-\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] + \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right]\right)\right) +$$

$$\frac{2}{11} b^2 (c + d x) \operatorname{ArcSinh}[c + d x] \left(11 \operatorname{ArcSinh}[c + d x] - 4 (c + d x) \sqrt{1 + (c + d x)^2}\right.$$

$$\left.\left.\operatorname{Hypergeometric2F1}\left[1, \frac{13}{4}, \frac{15}{4}, -(c + d x)^2\right]\right) +\right.$$

$$\left.\left(945 b^2 \pi (c + d x)^3 \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{13}{4}, \frac{13}{4}\right\}, \left\{\frac{15}{4}, \frac{17}{4}\right\}, -(c + d x)^2\right]\right)\right)$$

$$\left(512 \sqrt{2} \operatorname{Gamma}\left[\frac{15}{4}\right] \operatorname{Gamma}\left[\frac{17}{4}\right]\right)$$

Problem 237: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c e + d e x)^{5/2} (a + b \operatorname{ArcSinh}[c + d x])^2 dx$$

Optimal (type 5, 134 leaves, 3 steps) :

$$\frac{2 (e (c + d x))^{7/2} (a + b \operatorname{ArcSinh}[c + d x])^2}{7 d e} - \frac{1}{63 d e^2}$$

$$8 b (e (c + d x))^{9/2} (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{9}{4}, \frac{13}{4}, -(c + d x)^2\right] +$$

$$\frac{1}{693 d e^3} 16 b^2 (e (c + d x))^{11/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{11}{4}, \frac{11}{4}\right\}, \left\{\frac{13}{4}, \frac{15}{4}\right\}, -(c + d x)^2\right]$$

Result (type 5, 334 leaves) :

$$\begin{aligned}
& \frac{1}{6174 d} (e (c + d x))^{5/2} \\
& \left(1764 a^2 (c + d x) + 168 a b \left(- \frac{2 \sqrt{1 + (c + d x)^2} (-5 + 3 (c + d x)^2)}{(c + d x)^2} + 21 (c + d x) \operatorname{ArcSinh}[c + d x] - \right. \right. \\
& \left. \left. \frac{10 (-1)^{1/4} \sqrt{1 + (c + d x)^2} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c+d x}}\right], -1\right]}{(c + d x)^{7/2} \sqrt{1 + \frac{1}{(c+d x)^2}}} \right) + \right. \\
& \left. \frac{1}{(c + d x)^2} b^2 \left(-1336 (c + d x) + 1932 \sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x] - \right. \right. \\
& \left. \left. 1323 (c + d x) \operatorname{ArcSinh}[c + d x]^2 - 252 \operatorname{ArcSinh}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcSinh}[c + d x]] - \right. \right. \\
& \left. \left. 1680 \sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, -(c + d x)^2\right] + \right. \right. \\
& \left. \left. \left(210 \sqrt{2} \pi (c + d x) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, -(c + d x)^2\right] \right) / \right. \right. \\
& \left. \left. \left(\operatorname{Gamma}\left[\frac{5}{4}\right] \operatorname{Gamma}\left[\frac{7}{4}\right] \right) + 72 \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] + \right. \right. \\
& \left. \left. 441 \operatorname{ArcSinh}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcSinh}[c + d x]] \right) \right)
\end{aligned}$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int (c e + d e x)^{3/2} (a + b \operatorname{ArcSinh}[c + d x])^2 dx$$

Optimal (type 5, 134 leaves, 3 steps) :

$$\begin{aligned}
& \frac{2 (e (c + d x))^{5/2} (a + b \operatorname{ArcSinh}[c + d x])^2}{5 d e} - \frac{1}{35 d e^2} \\
& 8 b (e (c + d x))^{7/2} (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{4}, \frac{11}{4}, -(c + d x)^2\right] + \\
& \frac{1}{315 d e^3} 16 b^2 (e (c + d x))^{9/2} \operatorname{HypergeometricPFQ}\left[1, \frac{9}{4}, \frac{9}{4}, \left\{\frac{11}{4}, \frac{13}{4}\right\}, -(c + d x)^2\right]
\end{aligned}$$

Result (type 5, 251 leaves) :

$$\frac{1}{5 d} \left(e (c + d x) \right)^{3/2} \left(2 a^2 (c + d x) - \frac{8}{5} a b \sqrt{1 + (c + d x)^2} + 4 a b (c + d x) \operatorname{ArcSinh}[c + d x] + \frac{1}{5 (c + d x)^{3/2}} \right.$$

$$24 (-1)^{3/4} a b \left(-\operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] + \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] \right) + \frac{2}{7} b^2 (c + d x) \operatorname{ArcSinh}[c + d x]$$

$$\left(7 \operatorname{ArcSinh}[c + d x] - 4 (c + d x) \sqrt{1 + (c + d x)^2} \operatorname{Hypergeometric2F1}\left[1, \frac{9}{4}, \frac{11}{4}, -(c + d x)^2\right] \right) +$$

$$\left. \left(15 b^2 \pi (c + d x)^3 \operatorname{HypergeometricPFQ}\left[\{1, \frac{9}{4}, \frac{9}{4}\}, \{\frac{11}{4}, \frac{13}{4}\}, -(c + d x)^2\right] \right) \middle/ \left(32 \sqrt{2} \operatorname{Gamma}\left[\frac{11}{4}\right] \operatorname{Gamma}\left[\frac{13}{4}\right] \right) \right)$$

Problem 239: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcSinh}[c + d x])^2 dx$$

Optimal (type 5, 134 leaves, 3 steps):

$$\frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x])^2}{3 d e} - \frac{1}{15 d e^2}$$

$$8 b (e (c + d x))^{5/2} (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, -(c + d x)^2\right] +$$

$$\frac{1}{105 d e^3} 16 b^2 (e (c + d x))^{7/2} \operatorname{HypergeometricPFQ}\left[\{1, \frac{7}{4}, \frac{7}{4}\}, \{\frac{9}{4}, \frac{11}{4}\}, -(c + d x)^2\right]$$

Result (type 5, 276 leaves):

$$\begin{aligned}
& \frac{1}{27 d} \sqrt{e (c + d x)} \\
& \left(18 a^2 (c + d x) + 36 a b (c + d x) \operatorname{ArcSinh}[c + d x] - 24 b^2 \sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x] + \right. \\
& \quad 2 b^2 (c + d x) (8 + 9 \operatorname{ArcSinh}[c + d x]^2) - \left(24 a b \left(\sqrt{c + d x} + (c + d x)^{5/2} - \right. \right. \\
& \quad \left. \left. (-1)^{1/4} (c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c + d x}}\right], -1\right] \right) \right) / \\
& \quad \left(\sqrt{c + d x} \sqrt{1 + (c + d x)^2} \right) + 24 b^2 \sqrt{1 + (c + d x)^2} \operatorname{ArcSinh}[c + d x] \\
& \quad \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, -(c + d x)^2\right] - \\
& \quad \left. \left(3 \sqrt{2} b^2 \pi (c + d x) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, -(c + d x)^2\right] \right) \right) / \\
& \quad \left(\operatorname{Gamma}\left[\frac{5}{4}\right] \operatorname{Gamma}\left[\frac{7}{4}\right] \right)
\end{aligned}$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^2}{\sqrt{c e + d e x}} dx$$

Optimal (type 5, 132 leaves, 3 steps):

$$\begin{aligned}
& \frac{2 \sqrt{e (c + d x)} (a + b \operatorname{ArcSinh}[c + d x])^2}{d e} - \frac{1}{3 d e^2} \\
& 8 b (e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -(c + d x)^2\right] + \\
& \frac{1}{15 d e^3} 16 b^2 (e (c + d x))^{5/2} \operatorname{HypergeometricPFQ}\left[\{1, \frac{5}{4}, \frac{5}{4}\}, \{\frac{7}{4}, \frac{9}{4}\}, -(c + d x)^2\right]
\end{aligned}$$

Result (type 5, 223 leaves):

$$\frac{1}{12 d \sqrt{e (c + d x)} \text{Gamma}\left[\frac{7}{4}\right] \text{Gamma}\left[\frac{9}{4}\right]} \\ \left(3 \sqrt{2} b^2 \pi (c + d x)^3 \text{HypergeometricPFQ}\left[\{1, \frac{5}{4}, \frac{5}{4}\}, \{\frac{7}{4}, \frac{9}{4}\}, -(c + d x)^2\right] + \right. \\ \left. 8 \text{Gamma}\left[\frac{7}{4}\right] \text{Gamma}\left[\frac{9}{4}\right] \left(12 (-1)^{3/4} a b \sqrt{c + d x} \text{EllipticE}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] - \right. \right. \\ \left. \left. 12 (-1)^{3/4} a b \sqrt{c + d x} \text{EllipticF}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] + \right. \right. \\ \left. \left. (c + d x) \left(3 (a + b \text{ArcSinh}[c + d x])^2 - 2 b^2 \text{ArcSinh}[c + d x] \right. \right. \right. \\ \left. \left. \left. \text{Hypergeometric2F1}\left[1, \frac{5}{4}, \frac{7}{4}, -(c + d x)^2\right] \text{Sinh}[2 \text{ArcSinh}[c + d x]]\right)\right)\right)$$

Problem 241: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \text{ArcSinh}[c + d x])^2}{(c e + d e x)^{3/2}} dx$$

Optimal (type 5, 130 leaves, 3 steps):

$$-\frac{2 (a + b \text{ArcSinh}[c + d x])^2}{d e \sqrt{e (c + d x)}} + \frac{1}{d e^2} \\ 8 b \sqrt{e (c + d x)} (a + b \text{ArcSinh}[c + d x]) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -(c + d x)^2\right] - \\ \frac{1}{3 d e^3} 16 b^2 (e (c + d x))^{3/2} \text{HypergeometricPFQ}\left[\{\frac{3}{4}, \frac{3}{4}, 1\}, \{\frac{5}{4}, \frac{7}{4}\}, -(c + d x)^2\right]$$

Result (type 5, 224 leaves):

$$\begin{aligned}
& \frac{1}{d (e (c + d x))^{3/2}} \left(-2 a^2 (c + d x) + 2 a b (c + d x)^{3/2} \right. \\
& \left. - \frac{2 \operatorname{ArcSinh}[c + d x]}{\sqrt{c + d x}} + \frac{4 (-1)^{1/4} \sqrt{1 + (c + d x)^2} \operatorname{EllipticF}\left[\frac{i}{2} \operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c+d x}}\right], -1\right]}{(c + d x) \sqrt{1 + \frac{1}{(c+d x)^2}}} \right) + \\
& b^2 (c + d x) \left(- \left(\left(\sqrt{2} \pi (c + d x)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, -(c + d x)^2\right] \right) \right. \right. \\
& \left. \left. \left(\operatorname{Gamma}\left[\frac{5}{4}\right] \operatorname{Gamma}\left[\frac{7}{4}\right] \right) - 2 \operatorname{ArcSinh}[c + d x] \right. \right. \\
& \left. \left. \left(\operatorname{ArcSinh}[c + d x] - 2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, -(c + d x)^2\right] \operatorname{Sinh}[2 \operatorname{ArcSinh}[c + d x]] \right) \right)
\end{aligned}$$

Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^2}{(c e + d e x)^{5/2}} dx$$

Optimal (type 5, 134 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 (a + b \operatorname{ArcSinh}[c + d x])^2}{3 d e (e (c + d x))^{3/2}} - \\
& \left(8 b (a + b \operatorname{ArcSinh}[c + d x]) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -(c + d x)^2\right] \right) / \left(3 d e^2 \sqrt{e (c + d x)} \right) + \\
& \frac{1}{3 d e^3} 16 b^2 \sqrt{e (c + d x)} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{4}, 1\right\}, \left\{\frac{3}{4}, \frac{5}{4}\right\}, -(c + d x)^2\right]
\end{aligned}$$

Result (type 5, 262 leaves):

$$\frac{1}{36 d e (e (c + d x))^{3/2}} \left(-24 a^2 + 48 a b \left(-\text{ArcSinh}[c + d x] - 2 (c + d x) \left(\sqrt{1 + (c + d x)^2} + (-1)^{3/4} \sqrt{c + d x} \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c + d x}\right], -1\right]\right)\right) + b^2 \left(32 (c + d x)^3 \sqrt{1 + (c + d x)^2} \text{ArcSinh}[c + d x] \text{Hypergeometric2F1}\left[1, \frac{5}{4}, \frac{7}{4}, -(c + d x)^2\right] - \left(3 \sqrt{2} \pi (c + d x)^4 \text{HypergeometricPFQ}\left[\{1, \frac{5}{4}, \frac{5}{4}\}, \{\frac{7}{4}, \frac{9}{4}\}, -(c + d x)^2\right] \right) \right) \right) \left(\text{Gamma}\left[\frac{7}{4}\right] \text{Gamma}\left[\frac{9}{4}\right] \right) - 24 \left(-8 (c + d x)^2 + \text{ArcSinh}[c + d x]^2 + 2 \text{ArcSinh}[c + d x] \text{Sinh}[2 \text{ArcSinh}[c + d x]] \right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \text{ArcSinh}[c + d x])^2}{(c e + d e x)^{7/2}} dx$$

Optimal (type 5, 134 leaves, 3 steps):

$$-\frac{2 (a + b \text{ArcSinh}[c + d x])^2}{5 d e (e (c + d x))^{5/2}} - \left(\frac{8 b (a + b \text{ArcSinh}[c + d x]) \text{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, -(c + d x)^2\right]}{15 d e^2 (e (c + d x))^{3/2}} - \frac{16 b^2 \text{HypergeometricPFQ}\left[\left\{-\frac{1}{4}, -\frac{1}{4}, 1\right\}, \left\{\frac{1}{4}, \frac{3}{4}\right\}, -(c + d x)^2\right]}{15 d e^3 \sqrt{e (c + d x)}} \right)$$

Result (type 5, 258 leaves):

$$\begin{aligned} & \frac{1}{15 d e (e (c + d x))^{5/2}} \left(-6 a^2 - 12 a b \text{ArcSinh}[c + d x] - \frac{1}{\sqrt{1 + (c + d x)^2}} 8 a b (c + d x) \right. \\ & \left. \left(1 + (c + d x)^2 + (-1)^{1/4} (c + d x)^{5/2} \sqrt{1 + \frac{1}{(c + d x)^2}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{c + d x}}\right], -1\right]\right) + \right. \\ & b^2 \left(8 - 6 \text{ArcSinh}[c + d x]^2 - 8 \cosh[2 \text{ArcSinh}[c + d x]] - \right. \\ & \left. 8 (c + d x)^3 \sqrt{1 + (c + d x)^2} \text{ArcSinh}[c + d x] \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{5}{4}, -(c + d x)^2\right] + \right. \\ & \left. \left(\sqrt{2} \pi (c + d x)^4 \text{HypergeometricPFQ}\left[\left\{\frac{3}{4}, \frac{3}{4}, 1\right\}, \left\{\frac{5}{4}, \frac{7}{4}\right\}, -(c + d x)^2\right] \right) \right) \left(\text{Gamma}\left[\frac{5}{4}\right] \text{Gamma}\left[\frac{7}{4}\right] \right) - 4 \text{ArcSinh}[c + d x] \text{Sinh}[2 \text{ArcSinh}[c + d x]] \right) \end{aligned}$$

Problem 245: Attempted integration timed out after 120 seconds.

$$\int (c e + d e x)^{5/2} (a + b \operatorname{ArcSinh}[c + d x])^3 dx$$

Optimal (type 8, 82 leaves, 2 steps):

$$\frac{2 (e (c + d x))^{7/2} (a + b \operatorname{ArcSinh}[c + d x])^3}{7 d e} - \frac{6 b \operatorname{Int}\left[\frac{(e (c + d x))^{7/2} (a + b \operatorname{ArcSinh}[c + d x])^2}{\sqrt{1 + (c + d x)^2}}, x\right]}{7 e}$$

Result (type 1, 1 leaves):

???

Problem 247: Attempted integration timed out after 120 seconds.

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcSinh}[c + d x])^3 dx$$

Optimal (type 8, 80 leaves, 2 steps):

$$\frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x])^3}{3 d e} - \frac{2 b \operatorname{Int}\left[\frac{(e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x])^2}{\sqrt{1 + (c + d x)^2}}, x\right]}{e}$$

Result (type 1, 1 leaves):

???

Problem 251: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^3}{(c e + d e x)^{7/2}} dx$$

Optimal (type 8, 82 leaves, 2 steps):

$$-\frac{2 (a + b \operatorname{ArcSinh}[c + d x])^3}{5 d e (e (c + d x))^{5/2}} + \frac{6 b \operatorname{Int}\left[\frac{(a + b \operatorname{ArcSinh}[c + d x])^2}{(e (c + d x))^{5/2} \sqrt{1 + (c + d x)^2}}, x\right]}{5 e}$$

Result (type 1, 1 leaves):

???

Problem 253: Attempted integration timed out after 120 seconds.

$$\int (c e + d e x)^{5/2} (a + b \operatorname{ArcSinh}[c + d x])^4 dx$$

Optimal (type 8, 82 leaves, 2 steps):

$$\frac{2 (e (c + d x))^{7/2} (a + b \operatorname{ArcSinh}[c + d x])^4}{7 d e} - \frac{8 b \operatorname{Int}\left[\frac{(e (c + d x))^{7/2} (a + b \operatorname{ArcSinh}[c + d x])^3}{\sqrt{1 + (c + d x)^2}}, x\right]}{7 e}$$

Result (type 1, 1 leaves) :

???

Problem 255: Attempted integration timed out after 120 seconds.

$$\int \sqrt{c e + d e x} (a + b \operatorname{ArcSinh}[c + d x])^4 dx$$

Optimal (type 8, 82 leaves, 2 steps) :

$$\frac{2 (e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x])^4}{3 d e} - \frac{8 b \operatorname{Int}\left[\frac{(e (c + d x))^{3/2} (a + b \operatorname{ArcSinh}[c + d x])^3}{\sqrt{1 + (c + d x)^2}}, x\right]}{3 e}$$

Result (type 1, 1 leaves) :

???

Problem 259: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcSinh}[c + d x])^4}{(c e + d e x)^{7/2}} dx$$

Optimal (type 8, 82 leaves, 2 steps) :

$$-\frac{2 (a + b \operatorname{ArcSinh}[c + d x])^4}{5 d e (e (c + d x))^{5/2}} + \frac{8 b \operatorname{Int}\left[\frac{(a + b \operatorname{ArcSinh}[c + d x])^3}{(e (c + d x))^{5/2} \sqrt{1 + (c + d x)^2}}, x\right]}{5 e}$$

Result (type 1, 1 leaves) :

???

Problem 284: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \operatorname{ArcSinh}[a x^2] dx$$

Optimal (type 4, 101 leaves, 4 steps) :

$$-\frac{2 x \sqrt{1 + a^2 x^4}}{9 a} + \frac{1}{3} x^3 \operatorname{ArcSinh}[a x^2] + \frac{\left(1 + a x^2\right) \sqrt{\frac{1 + a^2 x^4}{(1 + a x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{9 a^{3/2} \sqrt{1 + a^2 x^4}}$$

Result (type 4, 75 leaves) :

$$\frac{1}{9} \left(-\frac{2 (x + a^2 x^5)}{a \sqrt{1 + a^2 x^4}} + 3 x^3 \operatorname{ArcSinh}[a x^2] - \frac{2 \sqrt{\frac{i}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i}{a}} x\right], -1\right]}{a^2} \right)$$

Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcSinh}[a x^2] dx$$

Optimal (type 4, 162 leaves, 5 steps) :

$$\begin{aligned} & -\frac{2 x \sqrt{1+a^2 x^4}}{1+a x^2} + x \text{ArcSinh}[a x^2] + \frac{2 (1+a x^2) \sqrt{\frac{1+a^2 x^4}{(1+a x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{\sqrt{a} \sqrt{1+a^2 x^4}} \\ & \frac{(1+a x^2) \sqrt{\frac{1+a^2 x^4}{(1+a x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{\sqrt{a} \sqrt{1+a^2 x^4}} \end{aligned}$$

Result (type 4, 59 leaves) :

$$\begin{aligned} & x \text{ArcSinh}[a x^2] - \frac{1}{\sqrt{i a}} \\ & 2 \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{i a} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{i a} x\right], -1\right] \right) \end{aligned}$$

Problem 288: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcSinh}[a x^2]}{x^2} dx$$

Optimal (type 4, 75 leaves, 3 steps) :

$$\begin{aligned} & -\frac{\text{ArcSinh}[a x^2]}{x} + \frac{\sqrt{a} (1+a x^2) \sqrt{\frac{1+a^2 x^4}{(1+a x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{\sqrt{1+a^2 x^4}} \end{aligned}$$

Result (type 4, 42 leaves) :

$$-\frac{\text{ArcSinh}[a x^2] + 2 \sqrt{i a} x \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{i a} x\right], -1\right]}{x}$$

Problem 290: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcSinh}[a x^2]}{x^4} dx$$

Optimal (type 4, 197 leaves, 6 steps) :

$$\begin{aligned}
& -\frac{2 a \sqrt{1+a^2 x^4}}{3 x} + \frac{2 a^2 x \sqrt{1+a^2 x^4}}{3 (1+a x^2)} - \frac{\text{ArcSinh}[a x^2]}{3 x^3} - \\
& \frac{2 a^{3/2} (1+a x^2) \sqrt{\frac{1+a^2 x^4}{(1+a x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{3 \sqrt{1+a^2 x^4}} + \\
& \frac{a^{3/2} (1+a x^2) \sqrt{\frac{1+a^2 x^4}{(1+a x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{3 \sqrt{1+a^2 x^4}}
\end{aligned}$$

Result (type 4, 88 leaves):

$$\begin{aligned}
& \frac{1}{3} \left(-\frac{2 a \sqrt{1+a^2 x^4}}{x} - \frac{\text{ArcSinh}[a x^2]}{x^3} + \frac{1}{\sqrt{i a}} \right. \\
& \left. 2 a^2 \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{i a} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{i a} x\right], -1\right] \right) \right)
\end{aligned}$$

Problem 302: Result more than twice size of optimal antiderivative.

$$\int \text{ArcSinh}\left[\frac{a}{x}\right] dx$$

Optimal (type 3, 25 leaves, 5 steps):

$$x \text{ArcCsch}\left[\frac{x}{a}\right] + a \text{ArcTanh}\left[\sqrt{1+\frac{a^2}{x^2}}\right]$$

Result (type 3, 77 leaves):

$$x \text{ArcSinh}\left[\frac{a}{x}\right] + \frac{a \sqrt{a^2+x^2} \left(-\text{Log}\left[1-\frac{x}{\sqrt{a^2+x^2}}\right] + \text{Log}\left[1+\frac{x}{\sqrt{a^2+x^2}}\right]\right)}{2 \sqrt{1+\frac{a^2}{x^2}} x}$$

Problem 311: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSinh}[a x^n]}{x} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$-\frac{\text{ArcSinh}[a x^n]^2}{2 n} + \frac{\text{ArcSinh}[a x^n] \text{Log}\left[1-e^{2 \text{ArcSinh}[a x^n]}\right]}{n} + \frac{\text{PolyLog}\left[2, e^{2 \text{ArcSinh}[a x^n]}\right]}{2 n}$$

Result (type 4, 128 leaves):

$$\begin{aligned} & \text{ArcSinh}[a x^n] \text{Log}[x] + \frac{1}{2 \sqrt{a^2} n} \\ & a \left(\text{ArcSinh}[\sqrt{a^2} x^n]^2 + 2 \text{ArcSinh}[\sqrt{a^2} x^n] \text{Log}[1 - e^{-2 \text{ArcSinh}[\sqrt{a^2} x^n]}] - \right. \\ & \left. 2 n \text{Log}[x] \text{Log}[\sqrt{a^2} x^n + \sqrt{1 + a^2 x^{2n}}] - \text{PolyLog}[2, e^{-2 \text{ArcSinh}[\sqrt{a^2} x^n]}] \right) \end{aligned}$$

Problem 328: Unable to integrate problem.

$$\int (a + i b \text{ArcSin}[1 - i d x^2])^{5/2} dx$$

Optimal (type 4, 348 leaves, 2 steps) :

$$\begin{aligned} & 15 b^2 x \sqrt{a + i b \text{ArcSin}[1 - i d x^2]} - \\ & \frac{5 b \sqrt{2 i d x^2 + d^2 x^4} (a + i b \text{ArcSin}[1 - i d x^2])^{3/2}}{d x} + x (a + i b \text{ArcSin}[1 - i d x^2])^{5/2} + \\ & \left. \left\{ 15 b^2 \sqrt{\pi} x \text{FresnelS}\left[\frac{\sqrt{-\frac{i}{b}} \sqrt{a + i b \text{ArcSin}[1 - i d x^2]}}{\sqrt{\pi}} \right] \left(\cosh\left[\frac{a}{2b}\right] + i \sinh\left[\frac{a}{2b}\right] \right) \right\} / \right. \\ & \left. \left(\sqrt{-\frac{i}{b}} \left(\cos\left[\frac{1}{2} \text{ArcSin}[1 - i d x^2]\right] - \sin\left[\frac{1}{2} \text{ArcSin}[1 - i d x^2]\right] \right) \right) - \right. \\ & \left. \left\{ 15 \sqrt{-\frac{i}{b}} b^3 \sqrt{\pi} x \text{FresnelC}\left[\frac{\sqrt{-\frac{i}{b}} \sqrt{a + i b \text{ArcSin}[1 - i d x^2]}}{\sqrt{\pi}} \right] \left(i \cosh\left[\frac{a}{2b}\right] + \sinh\left[\frac{a}{2b}\right] \right) \right\} / \right. \\ & \left. \left(\cos\left[\frac{1}{2} \text{ArcSin}[1 - i d x^2]\right] - \sin\left[\frac{1}{2} \text{ArcSin}[1 - i d x^2]\right] \right) \right) \end{aligned}$$

Result (type 8, 24 leaves) :

$$\int (a + i b \text{ArcSin}[1 - i d x^2])^{5/2} dx$$

Problem 329: Unable to integrate problem.

$$\int (a + i b \text{ArcSin}[1 - i d x^2])^{3/2} dx$$

Optimal (type 4, 312 leaves, 2 steps) :

$$\begin{aligned}
& - \frac{3 b \sqrt{2 i d x^2 + d^2 x^4} \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]}}{d x} + x (a + i b \operatorname{ArcSin}[1 - i d x^2])^{3/2} + \\
& \left(3 \sqrt{i b} b \sqrt{\pi} \times \operatorname{FresnelC}\left[\frac{\sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]}}{\sqrt{i b} \sqrt{\pi}} \right] \left(i \operatorname{Cosh}\left[\frac{a}{2 b}\right] - \operatorname{Sinh}\left[\frac{a}{2 b}\right] \right) \right) / \\
& \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] \right) - \\
& \frac{3 b^2 \sqrt{\pi} \times \operatorname{FresnelS}\left[\frac{\sqrt{a+i b \operatorname{ArcSin}[1-i d x^2]}}{\sqrt{i b} \sqrt{\pi}} \right] \left(\operatorname{Cosh}\left[\frac{a}{2 b}\right] - i \operatorname{Sinh}\left[\frac{a}{2 b}\right] \right)}{\sqrt{i b} \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] \right)}
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int (a + i b \operatorname{ArcSin}[1 - i d x^2])^{3/2} dx$$

Problem 330: Unable to integrate problem.

$$\int \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]} dx$$

Optimal (type 4, 263 leaves, 1 step):

$$\begin{aligned}
& x \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]} + \frac{\sqrt{\pi} \times \operatorname{FresnelS}\left[\frac{\sqrt{-\frac{i}{b}} \sqrt{a+i b \operatorname{ArcSin}[1-i d x^2]}}{\sqrt{\pi}} \right] \left(\operatorname{Cosh}\left[\frac{a}{2 b}\right] + i \operatorname{Sinh}\left[\frac{a}{2 b}\right] \right)}{\sqrt{-\frac{i}{b}} \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] \right)} - \\
& \left(\sqrt{-\frac{i}{b}} b \sqrt{\pi} \times \operatorname{FresnelC}\left[\frac{\sqrt{-\frac{i}{b}} \sqrt{a+i b \operatorname{ArcSin}[1-i d x^2]}}{\sqrt{\pi}} \right] \left(i \operatorname{Cosh}\left[\frac{a}{2 b}\right] + \operatorname{Sinh}\left[\frac{a}{2 b}\right] \right) \right) / \\
& \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2]\right] \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]} dx$$

Problem 332: Unable to integrate problem.

$$\int \frac{1}{(a + i b \operatorname{ArcSin}[1 - i d x^2])^{3/2}} dx$$

Optimal (type 4, 291 leaves, 1 step):

$$\begin{aligned}
& - \frac{\sqrt{2 \operatorname{i} d x^2 + d^2 x^4}}{b d x \sqrt{a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2]}} - \\
& \left(\left(-\frac{\operatorname{i}}{b} \right)^{3/2} \sqrt{\pi} x \operatorname{FresnelC} \left[\frac{\sqrt{-\frac{\operatorname{i}}{b}} \sqrt{a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2]}}{\sqrt{\pi}} \right] \left(\operatorname{Cosh} \left[\frac{a}{2b} \right] - \operatorname{i} \operatorname{Sinh} \left[\frac{a}{2b} \right] \right) \right) / \\
& \left(\operatorname{Cos} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] - \operatorname{Sin} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] \right) + \\
& \left(\left(-\frac{\operatorname{i}}{b} \right)^{3/2} \sqrt{\pi} x \operatorname{Fresnels} \left[\frac{\sqrt{-\frac{\operatorname{i}}{b}} \sqrt{a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2]}}{\sqrt{\pi}} \right] \left(\operatorname{Cosh} \left[\frac{a}{2b} \right] + \operatorname{i} \operatorname{Sinh} \left[\frac{a}{2b} \right] \right) \right) / \\
& \left(\operatorname{Cos} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] - \operatorname{Sin} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{(a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2])^{3/2}} dx$$

Problem 333: Unable to integrate problem.

$$\int \frac{1}{(a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2])^{5/2}} dx$$

Optimal (type 4, 326 leaves, 2 steps):

$$\begin{aligned}
& - \frac{\sqrt{2 \operatorname{i} d x^2 + d^2 x^4}}{3 b d x \left(a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right)^{3/2}} - \frac{x}{3 b^2 \sqrt{a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2]}} - \\
& \frac{\sqrt{\pi} x \operatorname{Fresnels} \left[\frac{\sqrt{a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2]}}{\sqrt{\operatorname{i} b} \sqrt{\pi}} \right] \left(\operatorname{Cosh} \left[\frac{a}{2b} \right] - \operatorname{i} \operatorname{Sinh} \left[\frac{a}{2b} \right] \right)}{3 \sqrt{\operatorname{i} b} b^2 \left(\operatorname{Cos} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] - \operatorname{Sin} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] \right)} - \\
& \frac{\sqrt{\pi} x \operatorname{FresnelC} \left[\frac{\sqrt{a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2]}}{\sqrt{\operatorname{i} b} \sqrt{\pi}} \right] \left(\operatorname{Cosh} \left[\frac{a}{2b} \right] + \operatorname{i} \operatorname{Sinh} \left[\frac{a}{2b} \right] \right)}{3 \sqrt{\operatorname{i} b} b^2 \left(\operatorname{Cos} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] - \operatorname{Sin} \left[\frac{1}{2} \operatorname{ArcSin}[1 - \operatorname{i} d x^2] \right] \right)}
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{(a + \operatorname{i} b \operatorname{ArcSin}[1 - \operatorname{i} d x^2])^{5/2}} dx$$

Problem 334: Unable to integrate problem.

$$\int \frac{1}{(a + i b \operatorname{ArcSin}[1 - i d x^2])^{7/2}} dx$$

Optimal (type 4, 389 leaves, 2 steps):

$$\begin{aligned}
& - \frac{\sqrt{2 i d x^2 + d^2 x^4}}{5 b d x (a + i b \operatorname{ArcSin}[1 - i d x^2])^{5/2}} - \\
& \frac{x}{15 b^2 (a + i b \operatorname{ArcSin}[1 - i d x^2])^{3/2}} - \frac{\sqrt{2 i d x^2 + d^2 x^4}}{15 b^3 d x \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]}} - \\
& \left(\left(-\frac{i}{b} \right)^{3/2} \sqrt{\pi} x \operatorname{FresnelC} \left[\frac{\sqrt{-\frac{i}{b}} \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]}}{\sqrt{\pi}} \right] \left(\cosh \left[\frac{a}{2 b} \right] - i \sinh \left[\frac{a}{2 b} \right] \right) \right) / \\
& \left(15 b^2 \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2] \right] \right) \right) + \\
& \left(\left(-\frac{i}{b} \right)^{3/2} \sqrt{\pi} x \operatorname{FresnelS} \left[\frac{\sqrt{-\frac{i}{b}} \sqrt{a + i b \operatorname{ArcSin}[1 - i d x^2]}}{\sqrt{\pi}} \right] \left(\cosh \left[\frac{a}{2 b} \right] + i \sinh \left[\frac{a}{2 b} \right] \right) \right) / \\
& \left(15 b^2 \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 - i d x^2] \right] \right) \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{(a + i b \operatorname{ArcSin}[1 - i d x^2])^{7/2}} dx$$

Problem 335: Unable to integrate problem.

$$\int (a - i b \operatorname{ArcSin}[1 + i d x^2])^{5/2} dx$$

Optimal (type 4, 348 leaves, 2 steps):

$$\begin{aligned}
& 15 b^2 x \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2]} - \\
& \frac{5 b \sqrt{-2 i d x^2 + d^2 x^4} (a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2])^{3/2}}{d x} + x (a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2])^{5/2} + \\
& \left(15 b^2 \sqrt{\pi} \times \operatorname{FresnelS}\left[\frac{\sqrt{\frac{i}{b}} \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2]}}{\sqrt{\pi}} \right] \left(\operatorname{Cosh}\left[\frac{a}{2b}\right] - \frac{i}{b} \operatorname{Sinh}\left[\frac{a}{2b}\right] \right) \right) / \\
& \left(\sqrt{\frac{i}{b}} \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] \right) \right) - \\
& \left(15 b^2 \sqrt{\pi} \times \operatorname{FresnelC}\left[\frac{\sqrt{\frac{i}{b}} \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2]}}{\sqrt{\pi}} \right] \left(\operatorname{Cosh}\left[\frac{a}{2b}\right] + \frac{i}{b} \operatorname{Sinh}\left[\frac{a}{2b}\right] \right) \right) / \\
& \left(\sqrt{\frac{i}{b}} \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] \right) \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int (a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2])^{5/2} dx$$

Problem 336: Unable to integrate problem.

$$\int (a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2])^{3/2} dx$$

Optimal (type 4, 310 leaves, 2 steps):

$$\begin{aligned}
& - \frac{3 b \sqrt{-2 i d x^2 + d^2 x^4} \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2]}}{d x} + x (a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2])^{3/2} - \\
& \frac{3 b^2 \sqrt{\pi} \times \operatorname{FresnelS}\left[\frac{\sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2]}}{\sqrt{-i b} \sqrt{\pi}} \right] \left(\operatorname{Cosh}\left[\frac{a}{2b}\right] + \frac{i}{b} \operatorname{Sinh}\left[\frac{a}{2b}\right] \right)}{\sqrt{-i b}} \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] \right) - \\
& \left(3 \sqrt{-i b} b \sqrt{\pi} \times \operatorname{FresnelC}\left[\frac{\sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + i d x^2]}}{\sqrt{-i b} \sqrt{\pi}} \right] \left(\frac{i}{b} \operatorname{Cosh}\left[\frac{a}{2b}\right] + \operatorname{Sinh}\left[\frac{a}{2b}\right] \right) \right) / \\
& \left(\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2]\right] \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int (a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2])^{3/2} dx$$

Problem 337: Unable to integrate problem.

$$\int \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]} dx$$

Optimal (type 4, 262 leaves, 1 step):

$$\begin{aligned} & x \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]} + \frac{\sqrt{\pi} \times \operatorname{FresnelS}\left[\frac{\sqrt{\frac{i}{b}} \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]}}{\sqrt{\pi}}\right] \left(\cosh\left[\frac{a}{2b}\right] - \frac{i}{2} \sinh\left[\frac{a}{2b}\right]\right)}{\sqrt{\frac{i}{b}} \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]\right] - \sin\left[\frac{1}{2} \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]\right]\right)} - \\ & \frac{\sqrt{\pi} \times \operatorname{FresnelC}\left[\frac{\sqrt{\frac{i}{b}} \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]}}{\sqrt{\pi}}\right] \left(\cosh\left[\frac{a}{2b}\right] + \frac{i}{2} \sinh\left[\frac{a}{2b}\right]\right)}{\sqrt{\frac{i}{b}} \left(\cos\left[\frac{1}{2} \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]\right] - \sin\left[\frac{1}{2} \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]\right]\right)} \end{aligned}$$

Result (type 8, 24 leaves):

$$\int \sqrt{a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2]} dx$$

Problem 339: Unable to integrate problem.

$$\int \frac{1}{(a - \frac{i}{b} b \operatorname{ArcSin}[1 + \frac{i}{d} d x^2])^{3/2}} dx$$

Optimal (type 4, 291 leaves, 1 step):

$$\begin{aligned}
& - \frac{\sqrt{-2 \operatorname{i} d x^2 + d^2 x^4}}{b d x \sqrt{a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2]}} + \\
& \left(\left(\frac{\operatorname{i}}{b} \right)^{3/2} \sqrt{\pi} \times \operatorname{FresnelS} \left[\frac{\sqrt{\frac{\operatorname{i}}{b}} \sqrt{a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2]}}{\sqrt{\pi}} \right] \left(\cosh \left[\frac{a}{2 b} \right] - \operatorname{i} \sinh \left[\frac{a}{2 b} \right] \right) \right) / \\
& \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] \right) - \\
& \left(\left(\frac{\operatorname{i}}{b} \right)^{3/2} \sqrt{\pi} \times \operatorname{FresnelC} \left[\frac{\sqrt{\frac{\operatorname{i}}{b}} \sqrt{a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2]}}{\sqrt{\pi}} \right] \left(\cosh \left[\frac{a}{2 b} \right] + \operatorname{i} \sinh \left[\frac{a}{2 b} \right] \right) \right) / \\
& \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{(a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2])^{3/2}} dx$$

Problem 340: Unable to integrate problem.

$$\int \frac{1}{(a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2])^{5/2}} dx$$

Optimal (type 4, 326 leaves, 2 steps):

$$\begin{aligned}
& - \frac{\sqrt{-2 \operatorname{i} d x^2 + d^2 x^4}}{3 b d x \left(a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right)^{3/2}} - \frac{x}{3 b^2 \sqrt{a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2]}} - \\
& \frac{\sqrt{\pi} \times \operatorname{FresnelS} \left[\frac{\sqrt{a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2]}}{\sqrt{-\operatorname{i} b} \sqrt{\pi}} \right] \left(\cosh \left[\frac{a}{2 b} \right] + \operatorname{i} \sinh \left[\frac{a}{2 b} \right] \right)}{3 \sqrt{-\operatorname{i} b} b^2 \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] \right)} - \\
& \left(\sqrt{-\operatorname{i} b} \sqrt{\pi} \times \operatorname{FresnelC} \left[\frac{\sqrt{a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2]}}{\sqrt{-\operatorname{i} b} \sqrt{\pi}} \right] \left(\operatorname{i} \cosh \left[\frac{a}{2 b} \right] + \sinh \left[\frac{a}{2 b} \right] \right) \right) / \\
& \left(3 b^3 \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 + \operatorname{i} d x^2] \right] \right) \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{(a - \operatorname{i} b \operatorname{ArcSin}[1 + \operatorname{i} d x^2])^{5/2}} dx$$

Problem 341: Unable to integrate problem.

$$\int \frac{1}{(a - i b \operatorname{ArcSin}[1 + i d x^2])^{7/2}} dx$$

Optimal (type 4, 389 leaves, 2 steps):

$$\begin{aligned}
& - \frac{\sqrt{-2 i d x^2 + d^2 x^4}}{5 b d x (a - i b \operatorname{ArcSin}[1 + i d x^2])^{5/2}} - \\
& \frac{x}{15 b^2 (a - i b \operatorname{ArcSin}[1 + i d x^2])^{3/2}} - \frac{\sqrt{-2 i d x^2 + d^2 x^4}}{15 b^3 d x \sqrt{a - i b \operatorname{ArcSin}[1 + i d x^2]}} - \\
& \left. \left(\left(\frac{i}{b} \right)^{3/2} \sqrt{\pi} x \operatorname{FresnelC} \left[\frac{\sqrt{\frac{i}{b}} \sqrt{a - i b \operatorname{ArcSin}[1 + i d x^2]}}{\sqrt{\pi}} \right] \left(\cosh \left[\frac{a}{2 b} \right] + i \sinh \left[\frac{a}{2 b} \right] \right) \right) \middle/ \right. \\
& \left. \left(15 b^2 \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2] \right] \right) + \right. \right. \\
& \left. \left. \sqrt{\frac{i}{b}} \sqrt{\pi} x \operatorname{FresnelS} \left[\frac{\sqrt{\frac{i}{b}} \sqrt{a - i b \operatorname{ArcSin}[1 + i d x^2]}}{\sqrt{\pi}} \right] \left(i \cosh \left[\frac{a}{2 b} \right] + \sinh \left[\frac{a}{2 b} \right] \right) \right) \middle/ \right. \\
& \left. \left(15 b^3 \left(\cos \left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2] \right] - \sin \left[\frac{1}{2} \operatorname{ArcSin}[1 + i d x^2] \right] \right) \right) \right)
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{(a - i b \operatorname{ArcSin}[1 + i d x^2])^{7/2}} dx$$

Problem 343: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcSinh} \left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}} \right] \right)^3}{1 - c^2 x^2} dx$$

Optimal (type 4, 261 leaves, 8 steps):

$$\frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^4}{4bc} - \frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} -$$

$$\frac{3b \left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c} +$$

$$\frac{3b^2 \left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c} - \frac{3b^3 \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{4c}$$

Result (type 8, 42 leaves) :

$$\int \frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Problem 344: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 195 leaves, 7 steps) :

$$\frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{3bc} - \frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} -$$

$$\frac{b \left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \frac{b^2 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c}$$

Result (type 8, 42 leaves) :

$$\int \frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Problem 345: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}{1 - c^2 x^2} dx$$

Optimal (type 4, 133 leaves, 6 steps) :

$$\frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{2bc} -$$

$$\frac{\left(a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} - \frac{b \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c}$$

Result (type 8, 40 leaves):

$$\int \frac{a + b \operatorname{ArcSinh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}{1 - c^2 x^2} dx$$

Problem 348: Attempted integration timed out after 120 seconds.

$$\int \operatorname{ArcSinh}[c e^{a+b x}] dx$$

Optimal (type 4, 76 leaves, 6 steps):

$$-\frac{\operatorname{ArcSinh}\left[c e^{a+b x}\right]^2}{2 b} + \frac{\operatorname{ArcSinh}\left[c e^{a+b x}\right] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[c e^{a+b x}\right]}\right]}{b} + \frac{\operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[c e^{a+b x}\right]}\right]}{2 b}$$

Result (type 1, 1 leaves):

???

Problem 368: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcSinh}\left[\frac{c}{a + b x}\right] dx$$

Optimal (type 3, 49 leaves, 6 steps):

$$\frac{(a + b x) \operatorname{ArcCsch}\left[\frac{a}{c} + \frac{b x}{c}\right]}{b} + \frac{c \operatorname{ArcTanh}\left[\sqrt{1 + \frac{1}{\left(\frac{a}{c} + \frac{b x}{c}\right)^2}}\right]}{b}$$

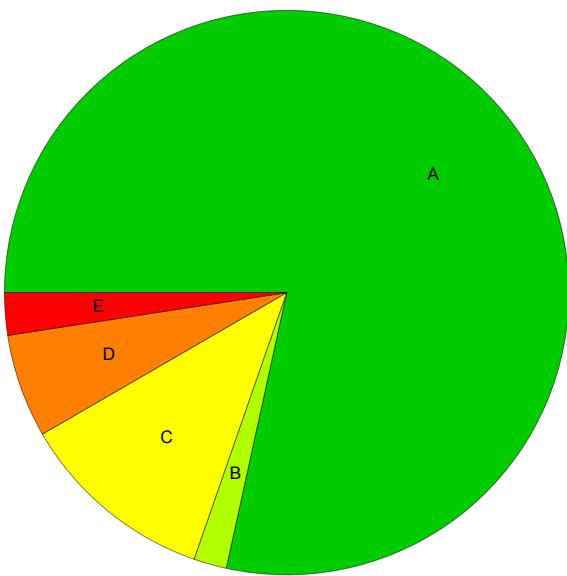
Result (type 3, 147 leaves):

$$x \operatorname{ArcSinh}\left[\frac{c}{a + b x}\right] +$$

$$\left((a + b x) \sqrt{\frac{a^2 + c^2 + 2 a b x + b^2 x^2}{(a + b x)^2}} \left(-a \operatorname{Log}[a + b x] + a \operatorname{Log}\left[c \left(c + \sqrt{a^2 + c^2 + 2 a b x + b^2 x^2}\right)\right] + c \operatorname{Log}\left[a + b x + \sqrt{a^2 + c^2 + 2 a b x + b^2 x^2}\right] \right) \right) / \left(b \sqrt{a^2 + c^2 + 2 a b x + b^2 x^2}\right)$$

Summary of Integration Test Results

371 integration problems



A - 291 optimal antiderivatives

B - 7 more than twice size of optimal antiderivatives

C - 42 unnecessarily complex antiderivatives

D - 22 unable to integrate problems

E - 9 integration timeouts